The Art of the Deal (and the Steal): Center-Local Bargaining and Distributive Politics under Electoral Autocracy

Allison Cuttner
Department of Political Science
University of Toronto
allison.cuttner@utoronto.ca

September 15, 2022

Abstract

How can the central government garner sufficient support given the control of local elites over their districts? Where should the center focus its public goods distribution and local elite payoffs across the state in order to maintain the regime? By circumventing local elites and appealing directly to the masses, a ruler can generate popular support in a geographic area without necessarily having to pay off a high-priced patron. With a multi-stage Rubinstein alternating offer bargaining program, I show how a central ruler can use investment in distribution to the masses to reduce the bargaining power of local elites while still using them to ensure the support of the district. I compare multiple institutional arrangements, public goods investment targeting across heterogeneous districts, and local elites with varying levels of control over their districts. My results show how surplus bargaining partners (i.e. unit proliferation) benefit the central government, how the center's investment in public goods varies with system design, and how weaker local elites increase public goods investment. I then use the case of Kenyan unit proliferation and public goods investment from the 1980s-2000s to illustrate the plausibility of my model.

Keywords: Bargaining, Distributive Politics, Clientelism, Unit Proliferation, Authoritarian Politics, Public Goods

Governmental leaders need the support or acquiescence of citizens to achieve and maintain power. In electoral autocracies and developing democracies, this can be achieved with sincere popularity among voters, fraud, or some combination thereof. Local elites[1] are advantaged, relative to the central government[2], in garnering the support of the citizens in their district. In some cases, these local elites are powerful patrons and traditional leaders whose clients will follow the political directives of the local leader due to existing relationships or expectations of reward (Koter, 2016). In other cases, local brokers simply have an informational advantage of local embeddedness: even without a hierarchical structure, they can recognize which voters in their district to target with vote buying and more easily monitor and punish defectors (Frye, Reuter and Szakonyi, 2019; Hidalgo and Nichter, 2016; Larreguy, Marshall and Querubin, 2016; Stokes et al., 2013). As local elites are thus advantaged in delivering their districts' support to the central leader, they are an invaluable component to the regime's maintenance of power.

When local support is needed to achieve and maintain power, the national leader could attempt to win local support by coopting these powerful local elites. In many authoritarian states and developing democracies in Eurasia, Africa, and Latin America, clientelistic structures permeate politics (Hale, 2014; Koter, 2016; Migdal, 1988). In situations when the local areas are organized into clientelist pyramids emanating from a single powerful patron (Hale, 2014), the central leader can use the structure to his advantage by only buying off the top elite. If there is a strong local elite who can whip votes, ensure effective policy implementation in the district, and generally support the needs of the regime at the local level, the leader can simply buy off this local elite in order to bring the entire locality into his fold. Similarly, a high-ranking broker with local relationships and information can be an effective "purchase" for the center: by coopting this single elite or local machine, the leader can ensure his victory in the district through vote buying or fraud. How exactly elites are included into the coalition in a way that ties their hands to supporting the regime can be

[1] I use local notable, elite, chief, and local leader interchangeably.

[2] I use central government, leader, ruler, and dictator interchangeably.

varied: positions in government (Blaydes, 2010), commitment to a dominant party (Reuter, 2017), or just more transfers than a challenger can offer (Groseclose and Snyder Jr, 1996; Bueno De Mesquita et al., 2003). While needing to buy off only one person at the top of the district "pyramid" may be most efficient, the local leader that has a tight grip on her district has a strong bargaining position and may drive up the "price" that she can charge the dictator in exchange for her support.

For a central leader trying to extract resources for himself, reliance on local power brokers can be expensive. By appealing directly to the masses, a ruler can generate popular support in a geographic area without having to pay off a high-priced patron. While electoral autocracies are not often recognized for their public goods distribution, mass coalition building through distributive politics is common across the developing world (Stokes et al., 2013). Turning out the vote with subsidies, transfers, and other benefits to mobilize supporters (Kasara, 2007) is common. Leaders may in fact pursue public policies that benefit large portions of the population such as land reform (Albertus, 2015) or more targeted benefits that can only be accessed by citizens in certain localities, ethnic groups, or industries. By using public goods and transfers to voters, the dictator can directly increase his own sincere popularity among voters, reducing the need for local elites to whip votes and commit fraud on his behalf.

How can the central government garner sufficient support given the control of local elites? Where should the center focus its public goods distribution and local elite payoffs across the state in order to maintain the regime? I show how a central ruler can use costly distribution to the masses to reduce the bargaining power of local elites. Investment in the infrastructure of mass politics allows a dictator to build a cheaper coalition of local elites because of the indirect effect of the dictator's popularity with citizens on the elites' bargaining strength, regardless of whether the dictator actually bypasses the local elites. My results show that while investment in mass politics supports the dictator's bargaining position relative to the local elite, it reduces the mutual surplus (i.e. efficiency relative to no elite) that using the elite intermediary provides. Thus there is a limit to how much costly investment in public goods the dictator will make when coopting the local leader is an option. Furthermore, the institutional environs of the center-local bargain drastically affects the dictator's popularity investment and resulting elite bargain. In a multi-district environment, the dictator's ability to play elites off of each other to create a minimal winning coalition further reduces the bargaining power of local elites. Thus unit proliferation, in which the central leader will have more potential bargaining partners, benefits the center. Where the dictator should invest in mass redistribution–districts in which he is already popular (core) or districts in which he is less popular (swing)– depends on how many districts are needed to maintain power.

I use the case of Kenya under two presidents– Jomo Kenyatta and Daniel Arap Moi– to demonstrate the plausibility of my model. In addition to qualitative evidence of strategic unit proliferation to increase the power of the central government relative to local (provincial) power brokers, I use evidence of public goods distribution from the center through road expenditures and center-local elite transfers though the *harambee* system of local development funding for education infrastructure. I compare the patterns of investments in both types of transfers across the presidencies and the types of districts–swing, core, or opposition– based on coethnic alignments and find that public goods and local elite expenditures are positively correlated in districts targeted for cooptation; these relationships are exacerbated in the multi-party electoral era, when district support for the incumbent was more salient.

This study has implications for not only the power of the central government vis-a-vis local strongmen but the distribution of public and nonprogrammatic goods. Graft and corruption, the representation of certain districts in the central government, and other benefits that the local elites can demand from the center directly depend on the dictator's ability to build local support among voters and the institutional rules that define central-local relations. The model suggests a plausible explanation for the variation in targets in distributive politics –core supporters or swing districts– that has been subject to extensive debate (Cox and McCubbins, 1986; Shapiro et al., 2010). The structures of the center-local relations that incentivize mass popularity in the model, specifically subunanimous district requirements and weak local elites, can explain why some states turn to populism while others maintain clientilistic systems. Additionally, a dictator's use of investment in mass politics to increase his bargaining power while simultaneously coopting local elites to deliver their district's support (albeit at a lower price) may explain the outsized margins of voter fraud in areas where the dictator is already popular (Rundlett and Svolik, 2016).

Building and Maintaining Local Support

The strength, legitimacy, and authority of local elites varies substantially across states and even within countries. Politicians can mobilize voters "...by indirectly working through electoral intermediaries: local leaders who command moral authority, control resources and can influence the electoral behavior of their dependents" (Koter, 2016, 17). These local elites may derive their authority over voters through a religious order, such as Marabouts in Senegal (Koter, 2016), economic dominance in an area, as is the case of oligarchs in Eurasia (Hale, 2014) and prominent employers in Russia and Venezuela (Frye, Reuter and Szakonyi, 2019), or through official governmental position as a mayor of a large city, governor, or provincial administrator (Barkan and Chege, 1989; Bueno De Mesquita et al., 2003; Reuter, 2017). Regardless of how they derive their power, local elites can wield their influence over voters in their district to help or hurt the electoral chances of the central ruler. Even without the religious, economic, traditional, or moral authority to ensure the compliance of their voters, local leaders have advantages over the central government in ensuring citizens follow their directions.

Depending on the community, who is considered a local elite can vary. I follow Hassan's general definition as an individual "with popular mobilization capacity in a particular geographic area" (Hassan, 2020, 31). Whether this is a mayor, religious leader, bureaucratic administrator, cartel capo, or business mogul, the power that the local elite has over their patrons is what gives them value to the dictator, not their formal position. A local elite with a formal position in a bureaucratic hierarchy or ties to the center through a political party may be subject to central control (i.e. could be fired or expelled from the party), but this is unlikely to generate perfect compliance with the dictator's wishes: a less-embedded replacement would be less valuable for local support.

Local elites have information, monitoring, and punishment power that allow for micro-targeting of resources, favors, jobs, and other benefits (or punishments) within a district. This allows them to condition benefits for individuals in their geographic domain on the actual support the individual gives, whether through party membership, voting, or tacitly supporting regime policies (Stokes et al., 2013). The relationships that a local leader has with the voters themselves or the network of brokers and subordinates that they utilize to whip votes are invaluable and often irreplaceable as the ability to monitor their behavior and only reward those who follow the directives of the patron is paramount to the transaction (Larreguy, Marshall and Querubin, 2016; Stokes et al., 2013). Whether the electoral outcomes are a result of individual vote-buying or its variants (Gans-Morse, Mazzuca and Nichter, 2014), fraud at the polling place or aggregation level (Beber and Scacco, 2012), true moral or religious persuasion, or voters' heuristic belief that voting according to their patron's wishes will yield more preferable policies or development spending from the center (Carter and Hassan, 2021; Koter, 2016), the local elites' ability to deliver their locality makes them an important partner for the central leader to court. By using these intermediaries, however, the regime is giving both resources and the control of their distribution to local powerholders. The dictator might need to overpay (i.e. pay significantly more than the elite distributes to his clients) the local elites in his coalition, particularly those in large or pivotal districts, to keep them loyal. Without a sufficient number of districts to win a national-level election or a widespread lack of acquiescence that leads to a mass uprising, an incumbent ruler will lose power.

Outside of their support in an election, local elites are critical to policy and implementation (Sellers, Lidström and Bae, 2020). According to Sellers et al., the "relative stability of. . . authoritarian regimes often depends on bargains between the national state and local or regional elites... Provision of public goods like water, sewers, electricity, and other services, for instance, can build diffuse support for the state, its governing elites, and its policies" (Sellers, Lidström and Bae, 2020, 44). For example, Georges Frêche, the mayor of Montpellier, France, from 1977-2004, personalized city politics and used his relationships with national politicians and national position to garner subsidies and acceptance from the central government for his local development projects (Sellers, Lidström and Bae, 2020). Attempts at development and fundraising from any civic groups independent of the local elites and Mayor Frêche's clientelistic machine were "stifled" (Sellers, Lidström and Bae, 2020, 32).

How can the central government avoid being held hostage by strong local leaders who demand much in exchange for their base of voters? If there is not currently an elite intermediary, it is difficult to simply impose a loyal agent on a community. A regime lackey will not have the support and trust of locals. An effective intermediary needs to be locally embedded, respected, and powerful. If people like this already exist in the community, their bargaining power to be coopted by the regime makes them very expensive (Garfias and Sellars, 2018; Gerring et al., 2011). According to Koter, "as much as it might be tempting for political actors to manufacture intermediaries where they were hitherto lacking, such efforts are likely to fail. Strong relations between local leaders and their followers are not built overnight" (Koter, 2016, 39). Whether through ethnic ties to their community (Hassan, 2020) or through a long period of appointment to an official position in an area (Barkey, 1994), local elites that are embedded in the community over which they rule are better informed of their community's needs and political preferences and garner more trust and cooperation (Hassan, 2020).

If he cannot replace a local leader at the peak of a district hierarchy with his loyal lackey, the dictator's best option is to go around the local hierarchy and appeal directly to voters. The use of public goods and "pork" targeted at voters in different electoral districts has been widely studied in the literature (Golden and Min, 2013; Cox and McCubbins, 1986; Dixit and Londregan, 1996). Unlike the distribution that the local elite could undertake to buy (or legally persuade) voters, mass redistribution is inefficient as it fails to target the most persuadable voters on an individual level and nor monitor their resulting votes. Micro-targeting benefits to supporters or individuals who need a transfer in order to support the incumbent is not feasible without local intermediaries. Instead of a "carrot" of enticing voters to turnout, a local machine can also engage in electoral violence to prevent the challenger's supporters from voting (Hafner-Burton, Hyde and Jablonski, 2014; Rauschenbach and Paula, 2019). A local elite's expertise is needed to target political violence such that the incumbent's supporters vote while the challenger's stay home, otherwise a general suppression of the vote may not benefit the incumbent.

Without violence, infrastructure projects, subsidies for particular industries, or special treatment for certain ethnic groups are all ways that a dictator can build support among certain mass groups without a local elite intermediary. While these policies may have some level of excludability (i.e. you can only access the subsidy if you farm a particular crop), they are not excludable at the micro-level.

What the extant work on distributive politics fails to consider, however, is how the general distribution of public goods and more popular policies will affect the relationship between the dictator and local elite. Increasing his popularity among voters does not remove the local power structure from the local elite, it simply increases the dictator's likelihood of winning *without* the support of the local elite. I argue that the incumbent can use mass politics to increase his popularity with voters as a bargaining tactic: by making himself more likely to win his election without elite support, the dictator reduces the local elites' advantage in their districts and increases his own bargaining position. This means that the dictator can still coopt the local elite and utilize their influence and political machine to deliver the district, just at a lower price than if he had not increased his mass popularity. As I show below, this effect has a limit: as the dictator invests more in the district, the efficiency surplus that using the elite intermediary supplies is reduced. Thus the dictator will make a costly investment in mass politics to improve his bargaining position vis-a-vis the local elite, still utilize the local elite's machine at a reduced price of cooptation, and keep the investment sufficiently low that the technology of a local intermediary continues to yield a surplus. I compare multiple institutional environments to see how the number of potential bargaining partners and variation in the center's popularity affect the dictator's investment decisions.

As alluded to by the above discussion, there are some conditions that limit where I expect this theory to apply. First, there must be local elites who can deliver their geographic area electorally such that having their support must make the dictator more likely to win than he would without them, though how they generate the votes in their individual district does not matter. I focus on electoral autocratic regimes and developing/unconsolidated democracies because it is more likely that the local power broker can impact the district (possibly with fraud) in these cases. I assume these elites are neutral in that they do not share some innate affinity for the dictator, but they do not receive a counter offer from the challenger nor are they punished if they support a candidate who ultimately fails– if the dictator does not win the election, they are no better or worse off than before. The dictator cannot credibly threaten to have the local elite removed from their position of power in their district, whether it is formal or informal, so the only way for him to receive their support is by paying for it.

Second, there must be some policies or goods that the central government can provide to voters that the local elite cannot appreciably interfere with. If the local elite was in charge of the distribution of such a good, she could intercept the dictator's attempt to popularize himself with voters and (1) claim credit for the distribution and increase her own hold on the district, minimizing the effect the good has on the dictator's popularity, or (2) refuse to distribute the good altogether and prevent the dictator from altering his electoral chances and, therefore, his bargaining position.

Model

Consider a regime subdivided into I districts, $\{1, 2, ..., I\}$, which each have a local elite chief, e_i and a mass of voters modeled as a continuum. In order to stay in power, the dictator must win the support of a sufficient number of districts, $N \leq I$; different institutional configurations are defined by the number of districts, N, that the dictator must win in order to maintain power.[3] If the dictator is successful in winning at least the minimum number of districts, he will achieve a regime benefit R. To win districts, the dictator can sidestep the local elite and appeal directly to the masses of the district or meet the demands of the local elite and recruit her support in controlling the district. While the expression of support in a district could take a variety of forms (including simply paying taxes and acquiescing to the dictator's policies without unrest), I will model the district's support for the dictator (with or without the local elite as an ally) as an election.[4] If the dictator utilizes a local elite intermediary (meeting her cooptation demand), he will win the support of the district with certainty. Without the elite, the dictator may still win the district, but is subject to the uncertainty of the election (see below). The dictator will bargain with each elite individually with no reconsideration: once an agreement on the split of his regime benefit between himself and local elite has been reached, it will occur. If there is more than one district, nature chooses the order by which elites bargain with the dictator.

The dictator can improve his chances in the district election by investing in costly mass redistribution. In particular, he can use costly distribution to the masses to make himself more popular without having to go through the elite intermediary. Let ν_i be the amount of

[3] I will address requiring one of one, two of two, and one of two in the main text. Three district configurations are addressed solely in the appendix.

[4] In this context, the existence of an election does not imply that it is free nor fair, but a contest for support in which there is a positive relationship between the benefits the central government extends to citizens and the likelihood that the central government remains in power.

redistribution to district $i \in I$ and $c(\nu_i)$ its cost. The dictator chooses a level of investment $\nu \in [0, \infty)$ and pays cost $c(\nu_i)$ where $c'(\nu_i) > 0$, $c'' > 0$, and $c(0) = 0$. If he invests in multiple districts, the cost for his total investment is $C(\nu)$ where $C'(\nu) > 0$, $C'' > 0$, and $C(0) = 0$. The dictator's decision to invest occurs prior to any cooptation negotiations with the local elite. Let p_i be the cooptation payment (if any) paid to the elite leader of district i and the sum of cooptation payments $P = \Sigma p_i$. Let w be the number of districts the dictator wins. The dictator derives utility from the regime benefit R, which he receives if he wins sufficient districts $w \geq N$, less any cooptation cost from utilizing elite intermediaries (P) and cost of investment in mass redistribution $(C(\nu))$: he has no preference over ideological positioning or mass redistribution for ideological reasons. If he fails to win a sufficient number of districts$w < N$, with or without elite support, the dictator's reversion utility is 0.

$$U_D(\nu, P, w) = \begin{cases} R - P - C(\nu) & \text{if } w \geq N \\ 0 & \text{Otherwise} \end{cases}$$

Local elites derive utility from the amount of the regime benefit that they can extract from the dictator in exchange for delivering their district. If the dictator wins the district without making a deal with the local elite, the elite has a reversion utility of 0. I assume that the local elite does not incur any additional cost for capturing their district on the dictator's behalf: she has sufficient local power and knowledge to use the resources she receives from the dictator to convert the district efficiently. The elite only receives her promised cooptation price if the dictator is ultimately successful; because she is receiving a portion of the regime benefit, the local elite will get nothing if the dictator does not win enough districts to succeed.[5]

[5] I assume the local elite's fate in her own district is not tied to that of the dictator. She does not incur a cost for "backing the wrong horse“ if she delivers the district and the dictator ultimately loses. Any power and benefits she receives for being the district chief is unaltered by the dictator's election or lack thereof.

$$U_{e_i}(p_i, w) = \begin{cases} p_i & \text{if } w \geq N \\ 0 & \text{Otherwise} \end{cases}$$

The Election

The dictator (D) faces a non-strategic challenger (C) in an election in every district in I. The winner of a sufficient number of districts, $N \leq I$, receives the regime benefit R, which I normalize to 1. Each district i is characterized by a representative voter, m_i, who evaluates the dictator relative to the challenger based on the dictator's valence, which can be positive, where the dictator is more popular than the challenger, or negative, where the challenger is more popular. V_i denotes the valence of the dictator relative to the challenger for voter m_i; it is distributed uniformly, $V_i \sim U[-\beta_i + \alpha_i + \nu_i, \beta_i + \alpha_i + \nu_i]$ where $\beta_i > 0, \nu_i \geq 0$. The probability that the dictator wins is $1 - F_{V_i}(0)$. While β_i determines the dispersion of V_i, the mean of the distribution is determined by α_i which can be positive or negative. This is the natural valence the voter has for the dictator prior to any investment. If $\alpha_i > 0$, the dictator is more popular than the challenger; if $\alpha < 0$, the dictator is at a disadvantage; if $\alpha_i = 0$ the voter is neutral. Voter m_i will vote for the dictator D if and only if $V_i \geq 0$. I assume that the dictator's minimum *ex ante* probability of winning is non-zero: while his likelihood of winning a district may be extremely small, it is always positive.

$$U_{m_i} = \begin{cases} V_i & \text{Vote D} \\ 0 & \text{Vote C} \end{cases}$$

Note that as valence becomes more uncertain, the dictator's expectation of winning goes to $\frac{1}{2}$ regardless of the lean of the voter, α_i. The *ex ante* probability that the dictator wins the district is increasing in α_i as he is more popular as well as in ν_i, the investment he makes in appealing to the masses.

Bargaining Protocol and Sequence of Play

The game proceeds as follows. Nature chooses the order of elites with whom the dictator will bargain (if there is more than one district). The dictator then chooses a level of investment, ν_i in mass politics, at cost $c(\nu_i)$. The dictator then bargains with each district elite in the determined order. I utilize an alternating offer Rubinstein bargaining protocol with risk of breakdown.[6] Within a bargaining period, the dictator and local elite bargain over the division of the benefit the dictator achieves when the elite supports him, which is normalized to 1. The elite in the first bargaining position proposes a split of the regime benefit between herself and the dictator, which the dictator can accept or reject. Upon rejection, with probability $\delta \in [0, 1]$ the dictator can make a counter proposal that the elite may accept or reject, and with probability $1 - \delta$ bargaining breaks down and the dictator moves to the next bargaining position. If a proposed split is accepted (by either party), the dictator moves to the next bargaining position until the districts are exhausted. Once the dictator has bargained with the local elite in every district, regardless of whether an agreement was reached or bargaining broke down, the election is held: Nature independently draws V_i from F_{V_i} and the citizens m_i each cast their votes for the dictator or challenger. If the dictator wins sufficient districts $w \geq N$, he wins the regime benefit (normalized to 1), shares agreed upon portions with any coopted elites, and pays investment costs if investment was made.

For each district, the dictator's strategy is a vector of investment, a cooptation offer, and offer acceptance threshold, (ν_i, y_i, x_i') as a function of $\alpha_i, \beta_i, \delta, c(\nu_i), N$. Each elite's strategy is a pair of a cooptation offer and offer acceptance threshold, (x_i, y_i') which is a function of $\alpha_i, \beta_i, \delta, \nu_i, N$. Each player's acceptance threshold is the minimum amount he would be willing to accept to avoid continuing the bargain, as rejection comes with a risk of breakdown. To maximize their own utility, each player offers his bargaining partner their acceptance threshold: the share that makes them indifferent between accepting and rejecting the offer to continue the bargain. Thus the equilibrium accepted offer is the acceptance threshold.

[6] See formal appendix for general Rubinstein SPNE existence and uniqueness for this bargaining protocol.

Sequence Summary

- Nature selects district bargaining order
- Dictator chooses level of investment $\nu_i \geq 0$ for each district
- Dictator and Elite in first bargaining position engage in alternating offer bargain
  - Elite makes offer of x portion of the regime benefit to the dictator, keeping $1 - x$ for herself
  - Dictator can accept offer and move to the next Elite bargain or reject the offer
  - If rejected, bargaining continues with probability $\delta \in [0, 1]$ with the Dictator's offer to give y portion of the regime benefit to the Elite, keeping $1 - y$ for himself; with probability $1 - \delta$ bargaining breaks down and the Dictator moves to the next Elite
  - Bargaining continues with alternating offers until an offer is accepted or bargaining breaks down
- If $I > 1$, the Dictator moves to the next Elite and engages in alternating offer bargain described above until all districts are exhausted
- Nature draws V_i for each district and voters m_i cast their votes
- Payoffs are distributed according to who wins the regime benefit, investment costs, and agreed upon cooptation costs

Results

One District

I begin by considering $I = N = 1$. In order to achieve the regime benefit, the dictator must win the single district. He can stand for an election without the support of the elite and be subject to the uncertainty this entails, or coopt the district's local elite to win this district with certainty. Without an elite intermediary to bargain with, the dictator will achieve the regime benefit, normalized to 1, with the probability that he wins the district in the election without investment, $\gamma_1 = \frac{\alpha+\beta}{2\beta}$. Note that his expected utility is always increasing in his popularity in the district, but the effect of uncertainty depends on his popularity: when he is popular, his expected utility decreases in the uncertainty of the district; when he is unpopular, the dictator benefits from uncertainty in the district. In general, I denote the dictator's electoral chances (the probability he wins the election without the support of the local elite) as $\Omega_{1,1}$ and the dictator's chances of winning with the elite support is 1 as a coopted local elite will deliver her district with certainty. Note that when there is a single district to win, the probability the dictator wins the district and the probability he wins the regime overall is the same ($\gamma_1 = \Omega_{1,1}$), but this is **not** the case when there are multiple districts ($I > 1$). The marginal benefit of utilizing the local elite instead of standing for election on his own is $1 - \Omega_{1,1}$. I term this difference in utilizing the local elite relative to standing for election alone as the *bargaining surplus.* From bargaining with the elite, the dictator will retain a portion $\lambda \in [0, 1]$ of this surplus. In the case of a single district, $\lambda_{1,1} = \frac{\delta}{1+\delta}$.

For a general form, the dictator's electoral chances without the elite are his outside option, as that is his expected utility should bargaining fail. He would never accept a bargain that would leave him with less than his outside option– rejecting that offer would better his utility, even if bargaining failed. Thus it is the mutual surplus that is really being shared between the dictator and elites. The dictator's electoral outside option, the bargaining surplus, and the division of the surplus all depends on the institutional configuration–both how many districts are in the state (I) and how many the dictator must win in order to win the regime (N).

$$\underbrace{\Omega_{N,I}}_{\text{D Electoral Option}} + \underbrace{\lambda_{N,I}}_{\text{D Share}} \underbrace{(1-\Omega_{N,I})}_{\text{Bargaining Surplus}}$$

The remainder of the regime benefit is transferred to the elite as the price of her support. Note that the dictator never actually makes a counteroffer in equilibrium: he accepts the elite's initial offer, but that offer from the elite is taking the potential counteroffer into account.

Lemma 1. *When the dictator must win one of one districts, for any investment* $\nu \geq 0$*, the local elite makes him an offer of* $\Omega_{1,1} + \frac{\delta}{1+\delta}(1-\Omega_{1,1})$*, which he accepts. If he were to make a counteroffer, the dictator would offer the local elite* $\frac{\delta(1+\gamma_1)}{(1+\delta)(1-\gamma_1)}(1-\Omega_{1,1})$.

I term the equilibrium portion that the dictator takes as the *dictator's share*, which is a direct reflection of his bargaining power. As discussed above, the dictator is coming to this bargain in a privileged position: if the parties fail to reach an agreement, he maintains his electoral outside option. The bargaining surplus of coopting the elite intermediary–the difference between going it alone and winning with certainty, $1-\Omega_{1,1}$–is then split between the elite and the dictator according to $\lambda_{1,1}$. In the extreme case of $\delta = 1$, bargaining would continue infinitely and the parties would share this surplus 50:50. For $\delta < 1$, the portion of the surplus that the elite will take in equilibrium is larger than the dictator's portion.

The dictator's share is increasing in α, his *ex ante* popularity in the district. The dictator's popularity in the district directly affects his bargaining position with the local elite and thus his equilibrium share: as his popularity increases, his outside option, $\Omega_{1,1}$ increases. Note, however, that the bargaining surplus that the parties share, $1-\Omega_{1,1}$, is *decreasing* in α. As the dictator is more likely to win his election without elite support, the benefit that the elite provides is lessened. Thus while the dictator's outside option grows in his *ex ante* popularity, the surplus from using the elite decreases. For the single district case, the effect of the outside option dominates the decreasing surplus: the dictator's overall utility is increasing in his popularity.

The dictator has the option to actively increase his outside option of his unsupported election probability by investing in mass politics, but at a cost. For any $\nu > 0$, the voter's valence evaluation of the dictator has a positive shift, increasing his chances of being elected without elite support. He must pay a convex cost of $c(\nu)$ for this investment. Recall that the dictator makes his investment decision prior to bargaining with any elites and will thus pay the cost of mass redistribution regardless of whether he stands for election unsupported or not.

Assumption 1. *The dictator's optimal investment will not yield a certain electoral outcome:* $max\{\frac{\alpha+\beta+2\beta\delta}{2\beta^2(1+\delta)^2-1}, \frac{\beta-\alpha}{1+\beta^2(1+\delta)^2}\} < \beta - \alpha$

To simplify the analysis of the dictator's optimal investment in mass politics, I assume that the costs of investment are sufficient that he will never make an investment so large that winning the election on his own is guaranteed. The formal threshold is derived from the dictator's maximal optimal investment and the boundary of the uniform distribution. What his maximal optimal investment is depends on parameter magnitudes. A more complete treatment of the dictator's investment in boundary cases where this assumption is violated (and the probability of winning without investment is zero) is in the appendix.

Proposition 1. *When the dictator must win one of one districts and anticipates the equilibrium elite offer of* $\Omega_{1,1} + \frac{\delta}{1+\delta}(1 - \Omega_{1,1})$*, he makes optimal investment* $\nu^*_{1,1} = c'^{-1}(\frac{1}{2\beta(1+\delta)})$

Corollary 1. *When the dictator must win one of one districts, anticipates the equilibrium elite offer of* $\Omega_{1,1} + \frac{\delta}{1+\delta}(1-\Omega_{1,1})$*, and* $c(\nu) = \frac{\nu^2}{2}$*, he makes optimal investment* $\nu^*_{1,1} = \frac{1}{2\beta(1+\delta)}$.

Note that, in equilibrium, the dictator is still coopting the elite, not actually utilizing the election. The investment in mass politics is an indirect instrument by which the dictator reduces the bargaining power of the elite by making his outside option more attractive, thereby increasing his utility. By increasing his mass election probability, the dictator makes a more lucrative deal with the local elite. The dictator's optimal investment is decreasing in the uncertainty of his reelection, β. Higher levels of electoral uncertainty reduce the impact of investment in mass politics on the dictator's outside option and, thus, the bargain he can strike with the local elite.

How does the technology of a local intermediary affect the dictator's optimal investment? If the elite intermediary was not an option, as would be the case without strong local party machines, chiefs, or brokers that are able to deliver the district, the dictator would make an optimal investment of $\nu^*_{1,1}(Mass) = c'^{-1}(\frac{1}{2\beta})$.[7] This optimal investment simply balances the direct increase in election probability that the dictator receives from mass investment with its cost, $c(\nu)$.

Proposition 2. *With no elite intermediary, expected probability of winning the regime benefit* γ_1, *and investment cost* $c(\nu)$, *the dictator makes investment* $\nu^*_{1,1}(Mass) = c'^{-1}(\frac{1}{2\beta})$.

Note that $\nu^*_{1,1}(Mass) > \nu^*_{1,1}(Elite)$: the optimal investment that the dictator makes without the use of an elite intermediary is greater than the optimal investment he makes when an elite bargain is available. Because investment is used to alter the bargaining power under the elite bargaining technology, it's effect on the dictator's utility is mediated by the bargain. Recall that though $\Omega_{1,1}$ is increasing in the dictator's investment in mass redistribution, the bargaining surplus of $1 - \Omega_{1,1}$ is *decreasing* in the investment. Thus the increase in the dictator's share that greater investment brings is mitigated by its negative effect on the surplus that the dictator and local elite share. When the dictator invests a lot in his popularity, he is no longer gaining as much from utilizing the elite intermediary.

Multiple Districts

A single district for the dictator to win and achieve the regime benefit is a baseline for the model: most states have multiple geographic areas over which to rule. While control over all districts might maximize tax revenue (Cederman and Girardin, 2010; Garfias and

[7] With the convex cost functional form assumption of $c(\nu) = \frac{\nu^2}{2}$, this is $\nu^*_{1,1}(Mass) = \frac{1}{2\beta}$.

Sellars, 2018), a regime may be comfortable in power without the support of every region. A multi-district state may incorporate districts which differ in a variety of ways including geography, distance to the capital, demography, and, most importantly, valence for the dictator. I consider a two district state in which the districts differ in three possible ways: (1) the dictator's ex ante popularity, (2) the uncertainty of the dictator's election without the support of the local elite, or (3) both popularity and uncertainty. There are two possible institutional configurations for a two-district state: either the dictator must win both districts to stay in office or he only needs one of two. The dictator now makes investment specific to a district. He can invest in both districts, one or the other, or neither, but he must make all investments prior to bargaining with the elites. As before, investment is costly; I now further assume a complementarity in costs between districts, χ, such that $c(\nu_1, \nu_2) = \frac{\nu_1^2}{2} + \frac{\nu_2^2}{2} + \chi\nu_1\nu_2$. Investing in a single district involves the cost of building infrastructure, bureaucracy, and moving resources to voters in the district; investing in two districts requires those bureaucrats, builders, and central government administrators to split their attention between multiple locations. Whether the dictator wants to invest in both districts (and how much) is influenced by how much added difficulty there is in working in two different districts simultaneously. As described above, nature chooses which elite the dictator bargains with first and, after that bargain is concluded either with an agreement or breakdown, the dictator can then bargain with the second elite before the election occurs.

Two of Two Districts

Consider a two-district state in which the dictator must win both in order to achieve the regime benefit. Let each district have distinct valence distributions for the dictator: $V_1 \sim U[-\beta_1 + \alpha_1 + \nu_1, \beta_1 + \alpha_1 + \nu_1]$ and $V_2 \sim U[-\beta_2 + \alpha_2 + \nu_2, \beta_2 + \alpha_2 + \nu_2]$.[8] As he must win both, the dictator's true electoral outside option of winning both districts without elite support is $\hat{\Omega}_{2,2} \equiv \gamma_1\gamma_2$.

[8]This includes the possibility that $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$ for completeness. See appendix.

Lemma 2. *When the dictator must win two of two districts, for any investment vector* $(\nu_1, \nu_2) \geq 0$*, each local elite* $i \in \{1, 2\}$ *makes him an offer of* $\gamma_i + \lambda_{1,1}(1-\gamma_i)$*, which he accepts. If he were to make a counteroffer, the dictator would offer each local elite* $\frac{\delta(1+\gamma_i)}{1+\delta}$*. The dictator's expected utility is* $\hat{\Omega}_{2,2} + \frac{\delta(\gamma_1(1-\gamma_2)+\gamma_2(1-\gamma_1))+\delta^2(1-\gamma_1\gamma_2)}{(1+\delta)^2(1-\gamma_1\gamma_2)}(1-\hat{\Omega}_{2,2})$

Note that which bargaining position the elites are in does not matter: they each get a share of the regime benefit according to the characteristics of their district, not the bargaining order. As above, the initial equilibrium offer from the each elite is immediately accepted. As γ_i is a function of the dictator's ex ante popularity in each district, his share is increasing in his ex ante popularity in each district. [9]

How does investment in mass politics change when there are two heterogeneous districts? To parameterize the cost of the dictator's investment in each district, I utilize the specific functional form of $c(\nu_1, \nu_2) = \frac{\nu_1^2}{2} + \frac{\nu_2^2}{2} - \chi\nu_1\nu_2$ where $\chi \geq 0$ indicates a complementarity in costs (i.e. costs are not purely independent of each other). If this complementarity is sufficiently low (χ is sufficiently small) the dictator will invest in both districts simultaneously; if the costs are too high, the dictator will only invest in one district.

Lemma 3. *Anticipating the elite equilibrium offers, the dictator will always make some investment in mass politics.*

Proposition 3. *If* $\chi < \hat{\chi}_{2,2} = 1 + \frac{1}{4\beta_1\beta_2(1+\delta)^2}$*, the dictator invests* $\hat{\nu}_1^*, \hat{\nu}_2^* > 0$ *in district 1 and district 2, respectively. If* $\chi \geq \hat{\chi}_{2,2}$*, the dictator will invest in one district. If* $\alpha_2 - \alpha_1 > (\beta_1 - \beta_2)(1 + 2\delta)$*, invest in district 1, otherwise invest in district 2.*

As long as the dictator faces a strictly interior probability of winning or losing (i.e. not 0 or 1, see appendix), the dictator will always make some investment in mass politics to support his bargaining position vis-a-vis the local elites. When the costs of investing in mass politics in one district increase the costs of investing in the other, the dictator may not always invest in both districts despite the fact that he needs both to win the regime.

[9] An extensive discussion of sequential bargaining with multiple homogeneous districts is in the appendix.

Higher uncertainty (β) in the district reduces the dictator's investment there as uncertainty reduces the effectiveness of his investment. If the cost complementarity (χ) is sufficiently low, however, the dictator will invest in both districts. The total investment that the dictator makes in mass redistribution in both districts, $\hat{\nu}_1^* + \hat{\nu}_2^* = \frac{\alpha_1+\alpha_2+(1+2\delta)(\beta_1+\beta_2)}{4\beta_1\beta_2(1+\delta)^2(1+\chi)-1}$, is increasing the dictator's popularity in each district.[10] The specific amounts he puts toward each district, however, differ. In particular, investment in district one decreases in the dictator's popularity in that district and increases in the dictator's popularity in the other district (similarly, investment in district two is decreasing in the dictator's popularity there and increasing in his popularity in the other district). He puts more of his resources where is he less popular. Because he must ultimately win both districts in order to keep power, the dictator's returns to investment in one district are scaled by his likelihood of winning the other. Focusing more of his investment where he is less popular will thus maximize his returns to mass redistribution.

The reasoning behind this division of resources becomes clear when we consider the dictator's investment in a single district. When the frictions of working in both districts simultaneously generate too much costs (χ is high), the dictator must choose which single district to invest in. Note that which district he invests in is not a function of the bargaining order of each district. Instead, only his own potential electoral win probability in each district affects his investment decision. All else equal, the dictator will prefer to invest in the district in which he is less popular and more certain i.e. his investment will be more effective. Because these parameters are independent (a district can be more uncertain but he can be *ex ante* more popular), the magnitudes of the difference between the two districts matter when the dictator is simultaneously more popular and more uncertain in one district over the other. As can be seen from Proposition 3, the difference in the electoral uncertainties of the districts is weighted more than the difference in popularities. If district one's election is more uncertain, the dictator would have to be very unpopular in district one (relative to

[10]Full statements of optimal investments in each district are in the appendix.

district two) to induce him to invest there. Note that some investment will always occur; while a high complementarity in costs might reduce his investment, zero investment in either district is always a dominated strategy for the dictator.

One of Two Districts

The other possible institutional arrangement in a state with two heterogeneous districts is that the dictator need only win one of the two to win the regime benefit. Because he only needs one district, if the dictator achieves an agreement with the first elite, the second is completely superfluous: no offer of sharing will entice the dictator to accept. The second elite will be left with nothing and the dictator will only share the regime benefit with the first elite.

With an "extra" district, the dictator's bargaining position is affected in two ways. First, having a back up elite to bargain with if his negotiation with the first elite breaks down increases the dictator's share of the regime benefit that he can keep from the first elite. Second, the dictator's true electoral outside option–his probability of achieving the regime benefit without the support of either elite–increases as he only needs one of two. In particular, his electoral outside option is $\hat{\Omega}_{1,2} \equiv \gamma_1 + \gamma_2 - \gamma_1\gamma_2$. Both of these forces benefit the dictator.

Lemma 4. *In equilibrium, the elite in the second bargaining position will never receive a portion of the regime benefit greater than 0.*

Lemma 5. *When the dictator must win one of two districts, for any investment vector* $(\nu_1, \nu_2) \geq 0$*, the local elite in the first bargaining position makes him an offer of* $\hat{\Omega}_{1,2} + \lambda_{1,2}(1 - \hat{\Omega}_{1,2})$*, which he accepts. If he were to make a counteroffer, the dictator would offer this elite* $\frac{\delta(1+2\delta+\hat{\Omega}_{1,2})}{(1+\delta)^2(1-\gamma_1)(1-\gamma_2)}(1 - \hat{\Omega}_{1,2})$*. The local elite in the second bargaining position makes the dictator an offer of 1, which he accepts; if he were to make a counteroffer, the dictator would offer the second elite 0.*

Despite the exclusion of the superfluous elite in the share of the regime benefit, the existence of this elite still affects the bargain that the dictator can strike with his first local elite partner. Note that even if the second district in the bargaining order is is better for the dictator in terms of outside win probability ($\gamma_2 > \gamma_1$), the dictator will prefer to come to an agreement with the first elite as the superfluous bargaining partner will always make him better off. Further note that even if the dictator could choose the bargaining order (instead of nature), he will be indifferent between either bargaining order as his expected utility is the same and he has no preference over which local elite he coopts.

Lemma 6. *Anticipating the elite equilibrium offers, the dictator will always make some investment in mass politics.*

Proposition 4. *If* $\chi < \hat{\chi}_{1,2} = 1 - \frac{1}{4\beta_1\beta_2(1+\delta)^2}$, *the dictator invests* $\hat{\nu}_1^*, \hat{\nu}_2^* > 0$ *in district 1 and district 2, respectively. If* $\chi \geq \hat{\chi}_{1,2}$, *the dictator will invest in one district. If* $\alpha_1 - \alpha_2 > \beta_1 - \beta_2$ *invest in district 1, otherwise invest in district 2.*

As above, when his election probability is in $(0, 1)$, the dictator will always make some non-zero investment in mass politics. As in the two of two institutional configuration, if the cost complementarity (χ) is sufficiently low, the dictator will invest in both districts. The total investment that the dictator makes, $\hat{\nu}_1^* + \hat{\nu}_2^* = \frac{\beta_1 - \alpha_1 + \beta_2 - \alpha_2}{1 + 4\beta_1\beta_2(1+\delta)^2(1+\chi)}$, which is decreasing the dictator's popularity in each district.[11] Similar to the two of two institution, higher uncertainty (β) in the district reduces the dictator's investment there as uncertainty reduces the effectiveness of his investment.

Each targeted piece of this overall investment, however, has the opposite relationship with the dictator's popularity as the previous institutional configuration. Investment in district one increases in his popularity in district one and decreases in the dictator's ex ante popularity in district two. Similarly, investment in district two increases in his popularity there and decreases in his popularity in the other district. If costs are too high, the dictator

[11]Optimal investments are fully characterized in the appendix.

will choose a single district to invest in. The order in which he will bargain with the districts does not affect his investment choice, only his potential to win each district without the elite support matters. The dictator prefers to invest in the district in which he is already more popular and in which his chances are more certain. As these are independent, he could be both popular and have high electoral uncertainty in a single district. Where he invests in this case depends on his relative popularity and uncertainty between the districts, as can be seen in Proposition 4.

Note that which district the dictator invests in (or how his two district investment amount changes) differs under the two institutional configurations described here. When the dictator must win both districts (two of two), investment in mass politics is complementary, particularly if χ is low. When he must choose which district in which to invest, the dictator will invest in the district in which he is less popular. Because he needs both districts to win, he will use his resources in his weakest district. By contrast when only one of two districts is needed to win, investment in districts is substitutive: as he invests more in district 1, he will invest less in district 2 and vice versa. Both districts affect his expected utility through the bargain (and electoral outside option), so even investment in the district that he does not end up coopting helps the dictator's position. If costs are so high that he must choose which district to invest in, the one of two institutional configuration will lead him to invest in the district in which he is more popular. Instead of investing in his weakest district, he invests in his strongest district to make it as easy to win as possible *even if this is not the district that he ends up coopting through the elite bargain.* His bargain with the elites takes *both* district win probabilities into account, thus he will invest more where he is popular regardless of which elite he ends up coopting.

The thresholds in the cost complementarity, χ, at which the dictator moves from investing in both districts to just one also depends on the institutional configuration. The range of parameters for which he prefers to invest in both districts is greater when he must win two of two than one of two. Relative to his optimal investment scheme if there were no intermediaries (and he still needs one of two districts), the parameter range for which he invests in both districts is larger when he bargains with the elites. Investing in the superfluous district is more attractive when the elite intermediaries are available because investment not only increases his win probability but also increases his bargaining position. Note this is not to say that he invests more when there are elite intermediaries (he does not), but that he will invest in both districts, instead of just one, more frequently.

Discussion

The demand the elite makes of the dictator depends on whether the dictator has invested in mass politics as well as the dictator's incentives to reject her offers, which is a function of both his electoral prospects and the number of elites he has to bargain with.[12] This "outside option" of an unsupported election still yields the dictator a positive expected utility: he could win the election without the elite. The elite's utility if bargaining breaks down, however, is zero. While the elite is privileged in making the first offer, the dictator's non-zero outside option in case of breakdown gives him a stronger bargaining position.

How the dictator and elites negotiate is subject to both the electoral environment (how popular the dictator is and how uncertain his electoral prospects are), but also the *institutional environment* in which these bargains are taking place. In particular, how many districts does the dictator need to win the regime benefit? Even without the elite intermediaries, different configurations would yield the dictator different expected utilities: for example, the dictator's probability of winning two of two districts is less than his probability of winning one of two. Furthermore, how many districts the dictator needs to win determines whether there are "extra" districts. Not only is winning one of two districts more likely than two of two with the mass election, a one of two configuration also makes one of the elite intermediaries superfluous. The dictator doesn't *need* his first bargaining partner, he can

[12] All proofs are in the appendix, as well as a discussion of homogeneous multi-district configurations.

simply reject a bad offer and move on to the second elite as winning just the second district is sufficient to win.

Weak Local Elites

In the preceding analysis, the local elites were perfect, albeit costly, intermediaries: if the dictator successfully coopted a local elite, he would win that district with certainty. Despite the local information and voter mobilization (or coercion) tools that a local elite may have, he is unlikely to guarantee the district to the dictator with absolute certainty. I extend the basic framework to incorporate exogenous *ex ante* uncertainty in the local elites' ability to deliver their districts. This district-specific likelihood that the elite delivers her district upon being coopted is parameterized as $\eta_i \in (0, 1)$ for district i. Further, I parameterize the probability that the dictator wins the regime benefit with the support of the elites as $\kappa_{N,I}$. When the elites delivered their districts with certainty, $\kappa_{N,I} = 1$; with weak elites, $\kappa_{N,I}$ is a function of the elites' control over their districts as well as the dictator's likelihood of winning the district himself. A single district case is discussed first to introduce the extension; I then analyze two of two and one of two heterogeneous configurations with weak elites. Allowing the local elites' abilities to deliver their districts to vary, in addition to the dictator's unsupported electoral popularity, generates expectations regarding mass political investment in subnational units that vary not only by the popularity of the leader, but also the strength of the local machine, (un)democraticness of local election administration, or charismatic persuasion of a local chief or religious leader.

One District

The dictator's likelihood of winning the election alone is again $\gamma = \frac{\alpha+\beta}{2\beta}$, his outside option should the bargain fail. If the bargain succeeds and the dictator coopts the local elite, the local elite will deliver the district with probability η. Thus the dictator's chances of winning with elite support is $\kappa = \eta + \gamma(1 - \eta)$, as the dictator could still win through his electoral chances should the elite fail to deliver the district. As above, the first offer is accepted and the elite is paid his cooptation price (agreed upon through the bargain) regardless of whether he successfully delivers the district or not.[13]

Lemma 7. *When the dictator must win one of one districts, for any investment $\nu \geq 0$, the local elite makes him an offer of $\frac{\gamma+\delta(\eta+\gamma(1-\eta))}{(1+\delta)(\eta+\gamma(1-\eta))}$, which he accepts for an expected utility of $\Omega_{1,1} + \frac{\delta}{(1+\delta)}(\kappa - \Omega_{1,1})$. If he were to make a counteroffer, the dictator would offer the local elite $\frac{\delta\eta(1-\gamma)}{(1+\delta)(\eta+\gamma(1-\eta))}$.*

The dictator's share of the surplus (in his expected utility) is the same portion ($\lambda_{1,1}$) as under certain elite, however note that his expected utility is not equal to the amount he receives in the bargain (as it was with certain elites) because the elite could fail to deliver the district, the dictator could still lose the election, and be left with nothing. The (expected) split of the regime benefit that the dictator and local elite agree upon depends on the strength of the local elite: the more likely she is to deliver the district, the greater portion the local elite can demand (and the dictator's portion decreases). The dictator's overall expected utility, however, is increasing in the local elites' probability of delivering the district because, though he will have to pay her more, he is more likely to win office when she is stronger, thus the bargaining surplus is greater when the local elites is stronger.

I assume that the dictator's investment in mass politics solely affects his likelihood of winning his election and is orthogonal to the elites' probability of delivery. In other words, the dictator's investment does not make it "easier" for the elite to deliver the district if she is coopted.

Proposition 5. *When the dictator must win one of one districts and anticipates the equilibrium elite offer of $\frac{\gamma+\delta(\eta+\gamma(1-\eta))}{(1+\delta)(\eta+\gamma(1-\eta))}$, he makes optimal investment $\nu^*_{1,1} = c'^{-1}(\frac{1+\delta(1-\eta)}{2\beta(1+\delta)})$*

Corollary 2. *When the dictator must win one of one districts, anticipates the equilibrium elite offer of , and $c(\nu) = \frac{\nu^2}{2}$, he makes optimal investment $\nu^*_{1,1} = \frac{1+\delta(1-\eta)}{2\beta(1+\delta)}$.*

[13] I refer to this as unconditional payment. I explore conditional payment structures, both from the elite and dictator's perspectives, in the appendix.

This optimal investment is greater than the dictator's single district investment when the elite could deliver the district with certainty because investment now serves two purposes. First, as before, increasing his likelihood of winning the district without the elite increases his outside option and, therefore, bargaining position vis-a-vis the elite. Investment allows him to extract more from the regime benefit, giving the elite less. Second, investment directly increases his chances of achieving the regime benefit should the elite fail to deliver. Because there is a non-zero probability that, despite being coopted and paid, the elite fails to secure the district on his behalf, the dictator can use investment in his popularity as insurance against this failure.

Two of Two Districts

With two districts, let the likelihood that each elite delivers their district be $\eta_1, \eta_2 \in (0, 1)$ subscripted by nature's selection in the bargaining order (i.e. the first district in the bargaining order is district 1). It could be the case that $\eta_1 > \eta_2$, $\eta_1 < \eta_2$, or that each elite is equally likely to deliver their district. The dictator will win the regime benefit without coopting either elite with probability $\Omega_{2,2} = \gamma_1\gamma_2$. If the dictator secures the support of the elites, he will win with probability $\kappa = \eta_1\eta_2 + \gamma_1\gamma_2(1-\eta_1)(1-\eta_2) + \gamma_1(1-\eta_1)\eta_2 + \gamma_2(1-\eta_2)\eta_1$.

Lemma 8. *When the dictator must win two of two districts, for any investment vector* $(\nu_1, \nu_2) \geq 0$, *each local elite* $i \in \{1, 2\}$ *makes him an offer of* $\frac{\delta + \frac{\gamma_i}{\eta_i + \gamma_i(1-\eta_i)}}{1+\delta}$, *which he accepts for an expected utility of* $\Omega_{2,2} + \tilde{\lambda}_{2,2}(\kappa - \Omega_{2,2})$. *If he were to make a counteroffer, the dictator would offer each local elite* $\frac{\delta\eta_i(1-\gamma_i)}{(1+\delta)(\eta_i+\gamma_i(1-\eta_i))}$.

As above, the dictator's share of the regime benefit is increasing in his popularity in each district and decreasing in the elites' likelihood of delivering their districts– a stronger elite (higher η_i) can demand more for her support. The dictator's overall expected utility, however, is increasing in the elites' strength as it directly increases the likelihood that he wins the regime benefit, even if he has to pay more to coopt the elites. Note that the dictator's individual shares from his bargains with each elite are larger than the baseline in which the elites deliver with certainty: the dictator will not pay as much for the elites' support when it will not guarantee victory.

For multi-district investment, I utilize the same cost functional forms as the heterogeneous district baseline; the only difference is the dictator's objective function determining optimal investment as it now incorporates the possibility that he loses despite coopting the elites and the greater share of regime benefit he can garner, if successful, due to this uncertainty. As above, if the complementarity of costs is sufficiently low (χ is sufficiently small) the dictator will invest in both districts simultaneously; if the costs are too high, the dictator will only invest in one district.

Lemma 9. *Anticipating the elite equilibrium offers, the dictator will always make some investment in mass politics.*

Proposition 6. *If* $\chi < \tilde{\chi}_{2,2} = 1 + \frac{(-1+\delta(\eta_1-1))(-1+\delta(\eta_2-1))}{4\beta_1\beta_2(1+\delta)^2}$*, the dictator invests* $\tilde{\nu}_1^*, \tilde{\nu}_2^* > 0$ *in district 1 and district 2, respectively. If* $\chi \geq \tilde{\chi}_{2,2}$*, the dictator will invest in a single district.*

The full characterization of the optimal investments are in the appendix. Much like the heterogeneous two of two baseline, the dictator will invest more where he is less popular. How optimal investment is affected by the elites' ability to deliver their districts depends on the relative parameters. Recall that with weak elites, investment not only supports the dictator's bargaining position, but also helps him win through his own popularity if the elites fail to deliver their districts. In his individual bargain with each elite, the dictator's outside option is increasing in his investment in that district, his share of the surplus is increasing in investment, but the bargaining surplus itself is decreasing in investment. When investment is too costly (due to complementary costs) and the dictator only invests in one district, his investment is decreasing in the elite's strength in that district. In other words, if the elite is stronger he will invest less, but if the elite is weaker he will invest more. Which district the dictator will invest in is function of αs, βs ηs and δ (fully characterized in the appendix). All else equal but popularity ($\alpha_1 \neq \alpha_2$), he will invest where he is less popular, similar to the strong elite version. When his electoral prospects are the same in the two districts, the dictator will invest where the elite is weaker.

One of Two Districts

Because the local elites cannot deliver their districts with certainty, the second elite is no longer completely superfluous. Instead of only coopting the first elite and hoping that he can win his election or the elite delivers, the dictator will coopt both elites. Unlike the two of two district arrangement, however, the portions that each elite receive are asymmetric.

Lemma 10. *When the dictator must win one of two districts, for any investment vector* $(\nu_1, \nu_2) \geq 0$*, the local elites makes him offers of* x_1^*, x_2^**, which he accepts for an expected utility of* $\Omega_{1,2} + \tilde{\lambda}_{1,2}(\kappa - \Omega_{1,2})$*. If he were to make a counteroffer, the dictator would offer each local elite,* y_1^*, y_2^**, respectively.*

The equilibrium offers and the dictator's expected utility are fully characterized in the appendix. All else equal (the dictator's popularity in each district, the elites' strength), the portion of the regime benefit that the first elite will receive is much greater than what the second elite can get due to similar logic as the baseline one of two case. The marginal insurance that the second district offers the dictator when he only needs to win one is much lower than the first, so after he has already coopted the first district, the dictator need not offer the second elite much. However, this is not to say that the first elite will always receive more. If the second district's elite is much more likely to deliver their district than the first, his cooptation price will be higher than the first elite's. The dictator's baseline utility (before investment) is non-monotonic in the elites' strength. While stronger local elites means he is more likely to win and receive the regime benefit, the dictator would also need to pay them more.

Lemma 11. *Anticipating the elite equilibrium offers, the dictator will always make some investment in mass politics.*

Proposition 7. *If* $\chi < \tilde{\chi}_{1,2} = 1 - \frac{(\delta(\eta_1-1)-1)(\delta(\eta_2-1)-1)}{4\beta_1\beta_2(1+\delta)^2}$*, the dictator invests* $\tilde{\nu}_1^*, \tilde{\nu}_2^*$ *in district 1 and district 2, respectively. If* $\chi \geq \tilde{\chi}_{1,2}$*, the dictator will invest in one district. If* $\alpha_1 - \alpha_2 > \beta_1 - \beta_2$ *invest in district 1, otherwise in district 2.*

As before, the dictator will invest in both districts if the cost complementarity is sufficiently low. Similar to the baseline one of two district arrangement, the dictator will invest more where he is ahead. Each investment is increasing in the dictator's popularity in each district and decreasing in the elite's strength in each district.[14] If the cost complementarity is too expensive, the dictator will choose one district to invest in based on where he is farther ahead. The specific condition that characterizes this choice is the same as the baseline one of two comparison.

Weak Elites Discussion

The general patterns of cooptation and investment when local elites cannot deliver their districts with absolute certainty mirror the baseline, with a few important differences. First, investment in mass politics in a system with weak local elites now serves dual purposes: (1) improving the dictator's bargaining position vis-a-vis the elites as before, and (2) increasing the dictator's chances of winning the election outright as insurance against the elites' failure. What does it mean for elites to be weak? While local elites can whip votes through legal means, such as persuasion and their own popularity, district delivery would be more certain (and thus more lucrative for local chiefs) with electoral fraud, vote buying, and violence. Thus if we consider districts with weaker elites to be more democratic insofar as the election is more fair and expressive of voters' preferences, the implication that investment in mass politics is greatest where elites are weakest (all else equal) aligns with existing work on the relationship between democracy and public goods, albeit at a subnational level (Bueno De Mesquita et al., 2003; Deacon, 2009; Lake and Baum, 2001). Second, weaker elites are individually cheaper to coopt than strong elites. A weaker elite cannot extract as much of

[14] This is true for much of the parameter space unless $\alpha_i \to \beta_i$

the regime surplus from the dictator– without perfect control of her district, her bargaining position is weaker. This is not to say, however, that the dictator is better off with weak local elites. A weak elite not only reduces the bargaining surplus that can be shared (the pie is smaller) but also subjects the dictator's overall utility to uncertainty. Under perfect local control, the dictator was assured of reelection as long as he paid for sufficient districts; with weak elites, he could strike an acceptable bargain and make investments in his popularity yet still lose office. Lastly, the impact of a surplus bargaining partner is changes when elite delivery is uncertain. Recall with certain districts, a one of two institutional configuration let the dictator pay off only one elite (the second elite was coopted at price 0). Under the same configuration but with weak elites, coopting the second elite improves the dictator's chances of winning because the first elite may not deliver. Because of his value to the dictator, the second elite can extract some of the regime benefit.

In summary, the series of alternating-offer bargains with local elites under a variety of institutional configurations has generated predictions regarding where the leader will invest in his popularity among citizens: districts in which he is already popular or districts in which he is unpopular. While using mass politics to improve his own bargaining position vis-a-vis local elites benefits the dictator, the use of this strategy is limited not only by the outright costs of redistribution but the decreasing returns to popularity when elite intermediaries are available. The surplus in utility that using a local elite intermediary who can deliver their area with certainty provides is diminished as the dictator's electoral chances improve.

Kenya

Public goods investment and local elite cooptation in Kenya presents a unique opportunity to observe center-local informal transfers, but is also presents difficulties due to immutable preferences for coethnic candidates. Kenya has alternated between periods of electoral autocracy and democracy since its independence from the British in 1963 (Burgess et al., 2015;

Hornsby, 2013). The citizens of Kenya ascribe to over 40 different ethnic groups, the most populous of which are the Kikuyu, Luo, Luhya, Kalenjin, and Kamba (Hornsby, 2013). Like many electoral regimes in sub-saharan Africa, political rivalries between parties (during the periods in which multi-party elections were permitted) are often drawn on ethnic lines. Jomo Kenyatta, the anti-colonial activist and first president of Kenya, was a Kikuyu; a number of studies have found that the Kikuyu ethnic group was privileged in terms of government positions as well as public goods and transfers while Kenyatta was in power (Hornsby, 2013; Burgess et al., 2015; Kramon and Posner, 2016) . While the president held appointment powers for Provincial and District Commissioners in this period (and Kenyatta appointed many Kikuyu), local officials were largely left alone by the central government and allowed to develop strong local bases of support (Barkan and Chege, 1989).

When Kenyatta suddenly died of natural causes in 1978, his vice president, Daniel Arap Moi, ascended to the presidency. As a member of a different ethnic group, the Kalenjin, Moi had to contend with the local governments dominated by Kikuyu leaders (even where the district was not majority-Kikuyu) that had previously supported Kenyatta (Barkan and Chege, 1989). The ethnic coalition of Kikuyu and Kalenjin that characterized the Moi transition was relatively short-lived: an attempted coup led by Kikuyu officers in 1982 led Moi to exclude Kikuyus from his coalition thereafter (Burgess et al., 2015). Succumbing to both internal and international pressure, Kenya held its first multi-party elections in 1992, however with widespread electoral violence and allegations of fraud, neither the 1992 elections nor the subsequent 1997 elections should be considered free or fair (Hornsby, 2013). Moi stepped down from the presidency and his ruling party, KANU, was ousted from the presidency in the first alternation since its founding by Mwai Kibaki, a Kikuyu, and his opposition alliance, National Alliance of Rainbow Coalition (NARC).

Kenya is an excellent case to examine the plausibility of my theory for a few reasons. First and most importantly, the *harambee* system of localized funding, discussed in more detail below, is a unique opportunity to measure informal center-local elite cooptation transfers.

Unlike most publicly available center-local transfer data, which is subject to formal budget constraints in which the amount of money transferred is based on locality population, poverty rates, etc., *harambee* is informal and fully discretionary. Second, Kenya underwent a center-initiated devolution of local power from the provincial level to the district level that approximates an increase in bargaining partners for the dictator. Third, the salience of ethnicity for presidential vote share and distributive politics (Kramon and Posner, 2016; Hornsby, 2013) generates an obvious proxy for the central leader's popularity within a district. Ethnic politics, however, also present a difficulty for my theory and thus makes Kenya a hard test: as an immutable characteristic, the dictator's popularity will be more or less sensitive to public goods investment depending on the ethnic alignment of the district. In ethnic opposition districts, for example, no amount of public goods spending is likely to alter the median voter's assessment of the dictator. I thus distinguish between ethnically aligned, opposition, and non-aligned (swing) districts below based on whether the plurality of the district are the incumbent president's coethnics or not.

Unit Proliferation

How can the dictator move to a more beneficial institutional arrangement in which he has superfluous bargaining partners and can therefore keep more of the regime benefit? While direct institutional choice is outside the scope of the model, it is clear that while the dictator has an obvious preference for a non-unanimous, the local elites do not. Elites that are powerful enough to control their districts can cause a lot of problems for a dictator attempting to change the institutions of the state, whether that be through unit proliferation, centralization, or changes in electoral rules for higher offices.

This is precisely what happened in Kenya under Daniel Arap Moi in the 1980s. He wanted to reduce the power of provincial leaders, so first adjusted to a populist message to shore up his own popularity among citizens. He then implemented an institutional reform that devolved the power of provincial leaders to a lower level of government (districts) that were easier for him to control and limited the bargaining position of any one province/district (more bargaining partners). In 1983, Moi unilaterally instituted a massive institutional change across Kenya called "District Focus" (Barkan and Chege, 1989). Prior to Moi's ascension to the presidency,"Kenyatta [the previous president] governed via a stable clientelist system composed of regional and district-level leaders, who were permitted to establish their own political identities and local bases of power, provided they supported the President" (Barkan and Chege, 1989, 437). When the Kikuyu officers staged an unsuccessful coup in 1982, Moi was no longer tolerant of the strong Kikuyu provincial and district leaders, many of whom had been appointed by Kenyatta (Barkan and Chege, 1989; Hornsby, 2013). Instead, "Moi sought to bypass such leaders and their organizations to create directly his own personal following in the countryside" (Barkan and Chege, 1989, 437). The District Focus, which was announced in October 1982 and implemented in 1983, was advertised as a focus on rural development and more efficient local governance. Indeed, "Moi's populist mode of governance... had its intended effect of circumventing the influence of the most senior politicians of the Kenyatta era, especially those from the Central Province" (Barkan and Chege, 1989, 437-438). Despite the purported focus on equitable development, making the lower level district as the primary operational unit (rather than larger provinces) allowed Moi to "reduce the power of P.C.s [Provincial Commissioners]" (Barkan and Chege, 1989, 440).

What this example demonstrates is how devolution and unit proliferation has already been recognized as a strategy by which a dictator can strengthen himself relative to local elites. The model has demonstrated the two mechanisms by which this is the case: (1) the change in the dictator's likelihood of maintaining his regime through his own popularity without the use of the elites and (2) the bargaining strength of the dictator when there are superfluous bargaining partners to play off one another. Furthermore, in order to undermine the resistance of local leaders, who would likely oppose such a large institutional change that would disadvantage them, the dictator invests in his own popularity among citizens. By adopting populist policies and increasing his own "personal following," Moi supported the survival of his regime against the possible defections and difficulties that the local elites he subverted.

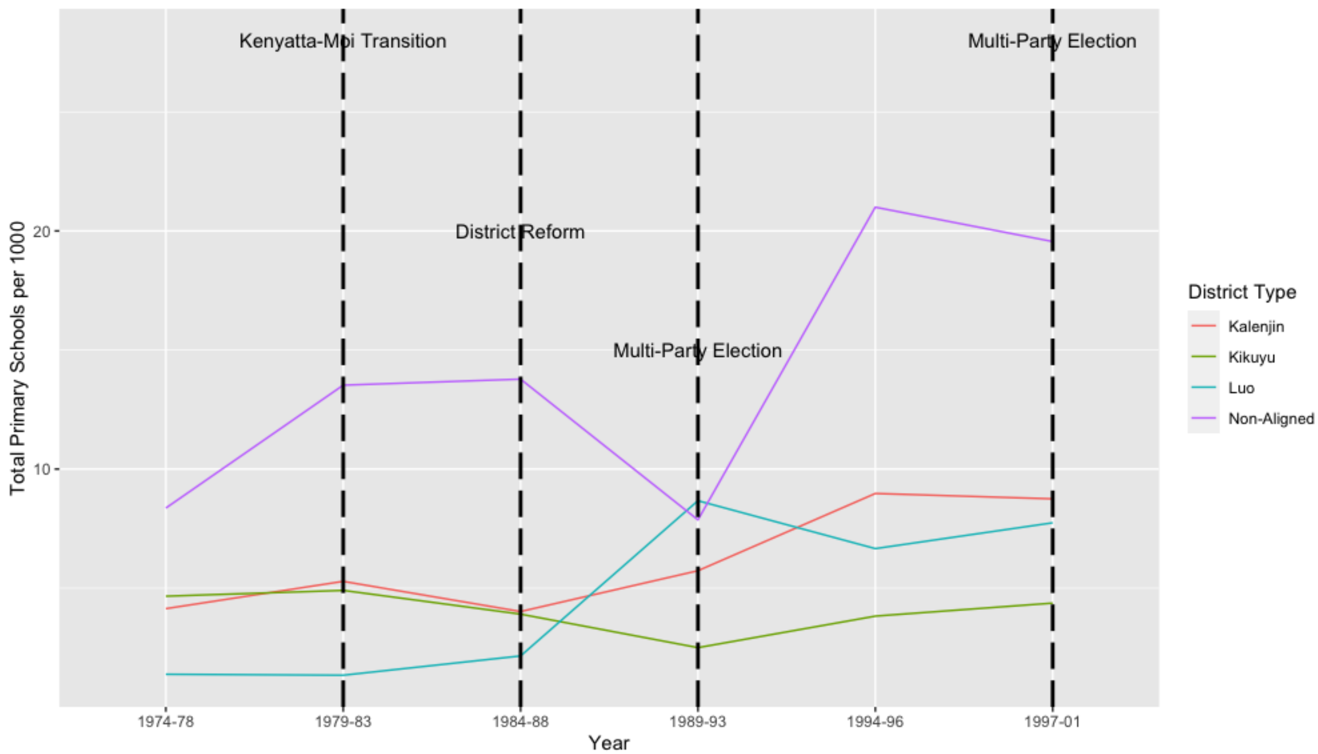
Local Elite Cooptation and Public Goods Investment

In line with existing literature on distributive politics, my theory predicts that the central government will use both public goods and elite transfers to increase their likelihood of winning an election. The relationship between these types of transfers, however, is not substitutive. Indeed, my model shows that the dictator will invest in his popularity through public goods (1) in districts he plans on coopting through elite transfers anyway, and (2) in districts in which he is already popular.[15] Observationally, as we cannot observe how much the local elite would cost if the dictator did not invest in his own district popularity through public goods, my theory predicts local elite transfers and public goods investment to be positively associated, especially in districts that the dictator needs to win. Opposition districts, then, have no expected relationship between transfers and public goods investment as the dictator need not waste resources on districts he has no chance of courting. I thus proxy for elite transfers and public goods investment in Kenya through the Kenyatta and Moi eras through population-normalized road expenditures and primary school construction by district.

A local development strategy known as *harambee*, meaning 'let's all pull together' (Hassan, 2020, 72), in which local community members and elites contribute to fund a project, began under Kenyatta's presidency as a strategy by which he could control local elites. Kenyatta provided donations to favored local elites and those "who did not receive visits from Kenyatta were less likely to meet the demands of their constituents" (Hassan, 2020, 92). Even after Kenyatta's tenure, presidents and central government ministers continued to make *harambee* contributions (Kramon and Posner, 2016). Education infrastructure, such

[15] Specifically in the one-of-two institutional configuration.

Figure 1: Total Schools per 1000 by Ethnicity



as school construction, are largely funded through *harambee* campaigns and "often receive substantial contributions from senior government officials" (Kramon and Posner, 2016, 36). While contributions to *harambee* funds are not necessarily lining a local elite's pocket directly, they (1) reduce the amount that the local elite must donate themselves (indirectly lining her pocket), and (2) improve the elite's standing with her community. Direct *harambee* contribution records are not available; however, the use of *harambee* contributions to fund education infrastructure makes school construction an effective proxy (Kramon and Posner, 2016). I use Kramon and Ponser's (2016) data on the number of primary schools per 1000 residents, gathered from District Development Reports at approximately five year intervals.[16]

[16]The apparent reduction in primary schools is due to rapid population increases, not bulldozing of schools. Unfortunately, population data by county is only available through the Kenyan census every 10 years, so de-normalizing this measure would require manifold assumptions regarding trends in population growth at the district level between census measurements. The reduction in schools per 1000 for non-aligned districts does indicate fewer education infrastructure resources per population.

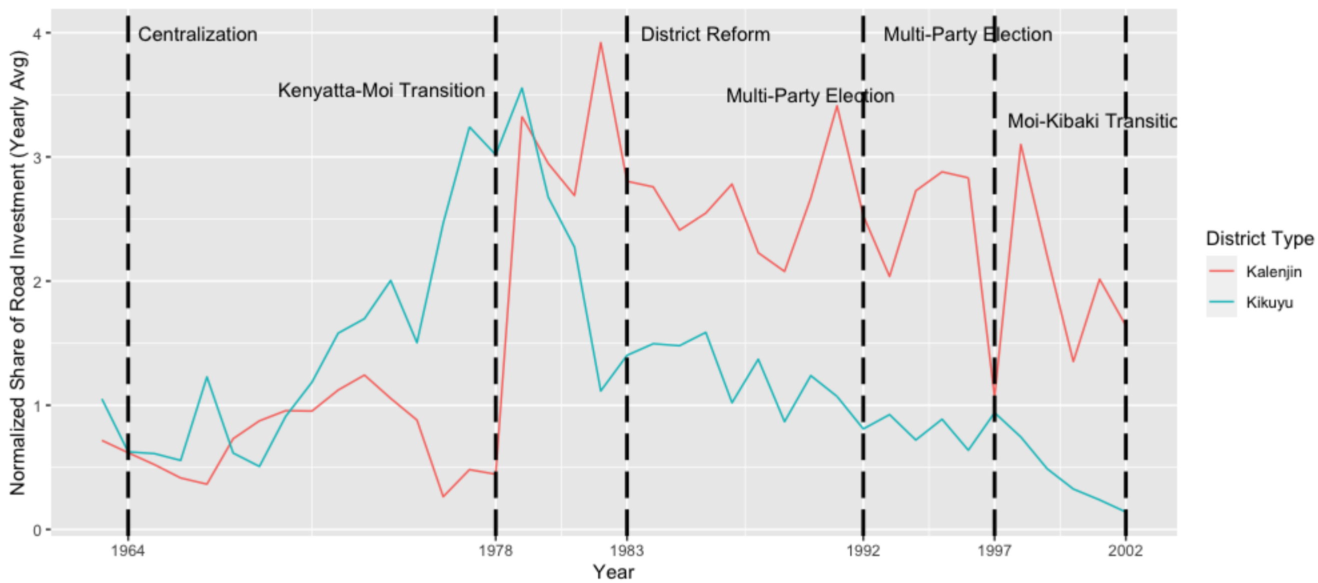
As expected, Kikuyu-majority districts after Moi takes power (and, in particular, after Kikuyu's are removed from his coalition after the 1982 coup attempt) are largely excluded. I separated Luo-plurality districts from other non-aligned districts (i.e. non-Kalenjin and non-Kikuyu) because two prominent Luo politicians, Oginga Odinga and his son Raila Odinga, ran for president in opposition to Moi in 1992 and 1997, respectively. Figure 1 suggests, however, that Luo districts follow a similar pattern to non-aligned districts in school construction, with large increases in the electoral era. How do these center-local elite transfers via *harambee* contributions compare to district-level public goods investment?

To measure the change in investment in mass politics by which the dictator increases his popularity in the district, I utilize road expenditures from Burgess et al. (2015). As the single largest development expenditure in the Annual Development Budget, road expenditures are not only an important aspect of Kenyan development, but are highly visible to citizens and very likely to alter their views of the central government. According to Burgess et al. (2015), "the Office of the President exercises strict oversight over road investment decisions" (1826). I utilize their measure of the normalized share of national road investment that a district received relative to the population of the district from 1963-1982.[17] With this normalization, a value of 1 implies that a district received road expenditures proportional to its population while a value greater than 1 indicates the expenditures are above the per capita average.

Figure 2 shows the average normalized road expenditures within each ethnic group's districts for Kikuyu and Kalenjin districts. With greater temporal coverage, this data shows the stark differences across the different presidential regimes. The trends follow the basic ethnic politics story explicated both in the distributive politics literature and studies of Kenya in particular (Burgess et al., 2015; Hornsby, 2013). Kikuyu districts were heavily favored under Kenyatta's (Kikuyu) presidency, with a sharp decline in resources corresponding with the Kikuyu coup attempt against Moi (Kalenjin). Kalenjin districts received massive amounts of expenditure across Moi's tenure, with large spikes in expenditures corresponding with Moi's

[17]Road Expenditures in district d and time t are calculated as $\frac{\frac{Exp_{d,t}}{Pop_{d,t}}}{\frac{Exp_{\text{National}}}{Pop_{\text{National}}}}$

Figure 2: Average Road Expenditures by Ethnicity, Kikuyu and Kalenjin Districts

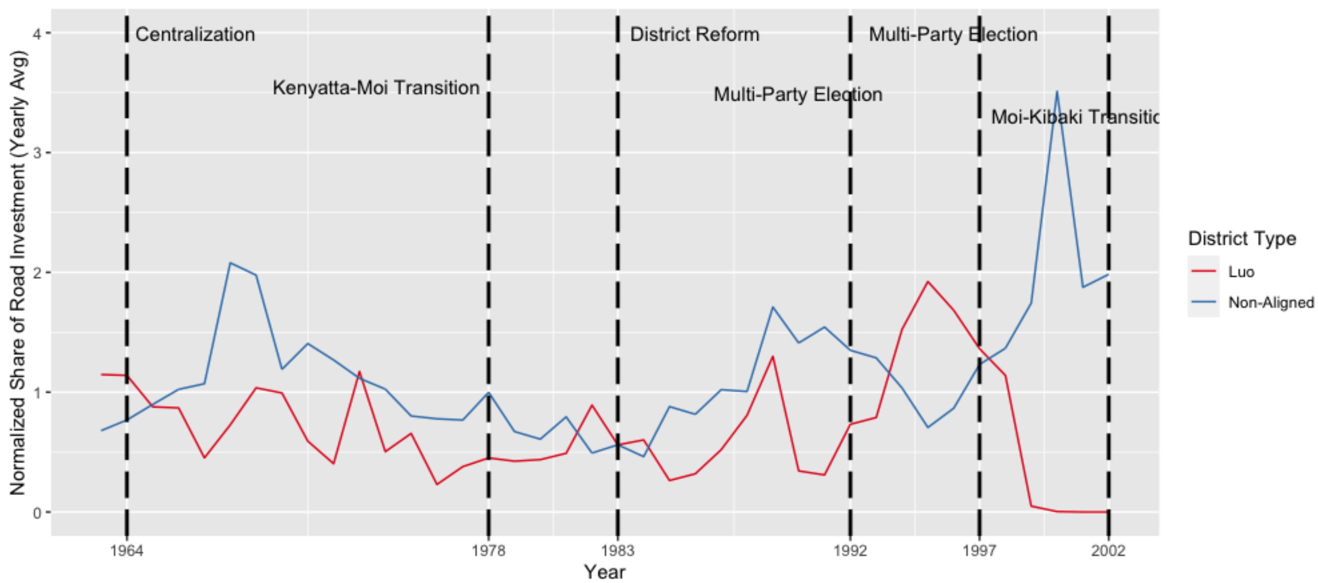


populist surge preceding District Reform as well as preceding each multi-party election in which Moi was running as the incumbent (1992 and 1997).

Figure 3 shows the average normalized road expenditures for Luo and non-aligned districts. While they received some public goods expenditures during the Kenyatta era, major public goods investments align with the electoral era. Non-aligned districts received boosts in road expenditures prior to the 1992 and 2002 elections while Luo district expenditures peak prior to the 1992 and 1997 elections. Moi's investment in Luo districts prior to the 1997 election appear to have paid off, despite a Luo candidate running for president: his vote share in the Luo-majority province of Nyanza increased from 15.2% in 1992 to 23.5% in 1997 (electoral results and discussion are in the Appendix).

Beyond the direction of public goods and local elite transfers via *harambee* contributions toward loyal coethnic and swing (non-aligned and Luo) districts, my theory's point of departure from previous literature on distributive politics is the relationship **between** elite transfers and public goods. I expect a positive correlation between primary schools and roads expenditures in districts that Moi is attempting to gain popularity **and** coopt: Kalen-

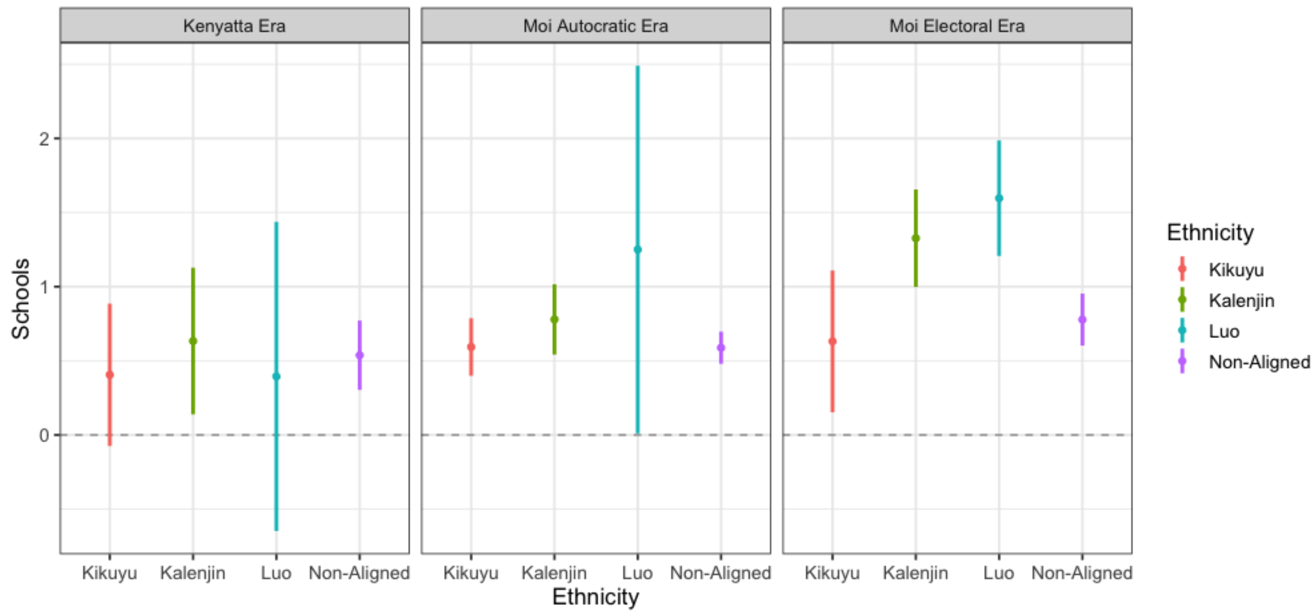
Figure 3: Average Road Expenditures by Ethnicity, Luo and Non-Aligned Districts



jin, Luo, and non-aligned districts. I do not expect a relationship between the two types of funding in Kikuyu districts under Moi's presidency. Furthermore, this is more likely to hold when district support is more vital to maintaining power: in particular, when Moi faces a multiparty election.

I regress primary schools on road expenditures and the district's ethnicity (Kikuyu, Kalenjin, Luo, and non-aligned) for three eras: Kenyatta's presidency (before 1978), Moi's autocratic era (1979-1988), and Moi's electoral era (1989-2001). The full results tables are in the Appendix. Figure 4 depicts the predicted schools per 1000 residents using the mean road expenditure by district type. As expected, the relationship between local elite transfers (schools) and public goods investment (roads) is not particularly strong and positive under Kenyatta, as he did not need to use public goods to increase his popularity as the father of the nation and exercised sufficient control over local areas through institutional centralization. Under Moi, the relationship between local cooptation transfers and public goods expenditure is more apparent, especially in the electoral era. In the electoral era, a one unit increase in normalized road expenditures is associated with a 1.42 increase in schools per

Figure 4: Predicted Schools by Mean Roads



1000 in Kalenjin districts, a 1.80 increase in schools per 1000 in Luo districts, and a 0.754 increase in schools per 1000 in non-aligned districts.

While data availability for *harambee* projects and county-level indicators in Kenya limit the power of these models, the relationship between public goods expenditure and local elite cooptation spending across the different institutional environs support the plausibility of my model. Future research on *harambee* contributions in Kenya as center-local elite transfers as well as other transfers in different state contexts will help support the important role that local elites play in distributive politics.

Conclusion

Under what conditions can a central incumbent undermine the power of local elites while still utilizing their embedded capacity on behalf of his regime? By extending a local-center bargaining framework to multiple heterogeneous districts and varying institutional rules, I show that while investment in his own popularity among citizens can improve his bargaining position, the positive effect that this has on his overall utility is diminishing. The greater benefit that using a local elite intermediary provides to the leader– the ability to monitor, reward, and punish at a targeted, micro level because of her local embeddedness– is mitigated as the leader becomes so popular he no longer needs the aforementioned strategies of generating support. Furthermore, where he invests in his electoral chances is a function of how difficult it is to redistribute to multiple districts simultaneously, his popularity and electoral uncertainty in each district, as well as the institutional arrangement he faces (unanimous or majority support).

While weaker local elites that cannot guarantee the support of their district are less expensive to coopt, the uncertainty that the bring to the election decreases both the bargaining surplus and the leader's overall expected utility. In accordance with much of the literature on public goods investment and regime type, I find that weaker local machines and patronage networks should be associated with greater investment in mass politics and central popularity to shore up the incumbent's chances of winning. The simultaneous use of investment in mass politics and local intermediaries to deliver a district's support could explain vote-buying and fraudulent electoral practices even when the dictator is popular (Rundlett and Svolik, 2016). Over the top returns– where the leader wins by such a large margin it is obviously irregular– could include sincere votes that he won through mass redistribution and true popularity among citizens as well as local brokers and patrons earning their share of the regime's power.

While negotiation between a dictator and the local elites he is buying off is difficult to observe, additional measurement of informal center-local transfers in contexts beyond Kenya would be beneficial for further testing of this theory. Future research on public goods distribution and its effects on electoral outcomes, especially in electoral autocracies and developing democracies, need to take local intermediaries into account.

References

Albertus, Michael. 2015. *Autocracy and redistribution*. Cambridge University Press.

Barkan, Joel D and Michael Chege. 1989. "Decentralising the state: district focus and the politics of reallocation in Kenya." *The Journal of Modern African Studies* 27(3):431–453.

Barkey, Karen. 1994. *Bandits and bureaucrats: The Ottoman route to state centralization*. Cornell University Press.

Beber, Bernd and Alexandra Scacco. 2012. "What the numbers say: A digit-based test for election fraud." *Political analysis* 20(2):211–234.

Blaydes, Lisa. 2010. *Elections and distributive politics in Mubarak's Egypt*. Cambridge University Press.

Bueno De Mesquita, Bruce, Alastair Smith, Randolph M Siverson and James D Morrow. 2003. *The logic of political survival*. MIT press.

Burgess, Robin, Remi Jedwab, Edward Miguel, Ameet Morjaria and Gerard Padró i Miquel. 2015. "The value of democracy: evidence from road building in Kenya." *American Economic Review* 105(6):1817–51.

Carter, Brett L and Mai Hassan. 2021. "Regional governance in divided societies: evidence from the Republic of Congo and Kenya." *The Journal of Politics* 83(1):000–000.

Cederman, Lars-Erik and Luc Girardin. 2010. "Growing sovereignty: modeling the shift from indirect to direct rule." *International Studies Quarterly* 54(1):27–48.

Cox, Gary W and Mathew D McCubbins. 1986. "Electoral politics as a redistributive game." *The Journal of Politics* 48(2):370–389.

Deacon, Robert T. 2009. "Public good provision under dictatorship and democracy." *Public choice* 139(1):241–262.

Dixit, Avinash and John Londregan. 1996. "The determinants of success of special interests in redistributive politics." *the Journal of Politics* 58(4):1132–1155.

Frye, Timothy, Ora John Reuter and David Szakonyi. 2019. "Hitting them with carrots: Voter intimidation and vote buying in Russia." *British Journal of Political Science* 49(3):857–881.

Gans-Morse, Jordan, Sebastian Mazzuca and Simeon Nichter. 2014. "Varieties of clientelism: Machine politics during elections." *American Journal of Political Science* 58(2):415–432.

Garfias, Francisco and Emily A Sellars. 2018. "From Conquest to Centralization: Domestic Conflict and the Transition to Direct Rule.".

Gerring, John, Daniel Ziblatt, Johan Van Gorp and Julián Arévalo. 2011. "An institutional theory of direct and indirect rule." *World Pol.* 63:377.

Golden, Miriam and Brian Min. 2013. "Distributive politics around the world." *Annual Review of Political Science* 16.

Groseclose, Tim and James M Snyder Jr. 1996. "Buying supermajorities." *American Political Science Review* pp. 303–315.

Hafner-Burton, Emilie M, Susan D Hyde and Ryan S Jablonski. 2014. "When do governments resort to election violence?" *British Journal of Political Science* 44(1):149–179.

Hale, Henry E. 2014. *Patronal politics: Eurasian regime dynamics in comparative perspective.* Cambridge University Press.

Hassan, Mai. 2020. *Regime threats and state solutions: Bureaucratic loyalty and embeddedness in Kenya.* Cambridge University Press.

Hidalgo, F Daniel and Simeon Nichter. 2016. "Voter buying: Shaping the electorate through clientelism." *American Journal of Political Science* 60(2):436–455.

Hornsby, Charles. 2013. *Kenya: A history since independence.* IB Tauris.

Kasara, Kimuli. 2007. "Tax me if you can: Ethnic geography, democracy, and the taxation of agriculture in Africa." *American political science review* pp. 159–172.

Koter, Dominika. 2016. *Beyond ethnic politics in Africa.* Cambridge University Press.

Kramon, Eric and Daniel N Posner. 2016. "Ethnic favoritism in education in Kenya." *Quarterly Journal of Political Science* 11(1):1–58.

Lake, David A and Matthew A Baum. 2001. "The invisible hand of democracy: political control and the provision of public services." *Comparative political studies* 34(6):587–621.

Larreguy, Horacio, John Marshall and Pablo Querubin. 2016. "Parties, brokers, and voter mobilization: How turnout buying depends upon the party's capacity to monitor brokers." *American Political Science Review* 110(1):160–179.

Migdal, Joel S. 1988. *Strong societies and weak states.* Princeton University Press.

Rauschenbach, Mascha and Katrin Paula. 2019. "Intimidating voters with violence and mobilizing them with clientelism." *Journal of Peace Research* 56(5):682–696.

Reuter, Ora John. 2017. *The origins of dominant parties: Building authoritarian institutions in post-Soviet Russia.* Cambridge University Press.

Rundlett, Ashlea and Milan W Svolik. 2016. "Deliver the vote! micromotives and macrobehavior in electoral fraud." *American Political Science Review* 110(1):180–197.

Sellers, Jefferey M, Anders Lidström and Yooil Bae. 2020. *Multilevel democracy: How local institutions and civil society shape the modern state.* Cambridge University Press.

Shapiro, Ian, Susan C Stokes, Elisabeth Jean Wood and Alexander S Kirshner. 2010. *Political representation.* Cambridge University Press.

Stokes, Susan C, Thad Dunning, Marcelo Nazareno and Valeria Brusco. 2013. *Brokers, voters, and clientelism: The puzzle of distributive politics*. Cambridge University Press.

Empirical Appendix

Table 1: Simple Regressions

	Dependent variable:			
	Primary Schools per 1000			
	(1)	(2)	(3)	(4)
Road Expenditure	−0.028	−0.014	−0.001	−0.017
	(0.085)	(0.010)	(0.008)	(0.017)
Kikuyu	−0.621	−0.625***	−0.271	−0.733**
	(0.459)	(0.209)	(0.205)	(0.324)
Luo	−0.265	0.160	−0.513	0.397
	(0.543)	(0.263)	(0.525)	(0.368)
Non-Aligned	−0.288	−0.558***	−0.181	−0.687***
	(0.354)	(0.164)	(0.165)	(0.238)
RoadExp:Kikuyu	0.076	0.020	0.014	0.006
	(0.087)	(0.018)	(0.013)	(0.053)
RoadExp:Luo	0.005	0.014	0.158	−0.019
	(0.179)	(0.037)	(0.175)	(0.047)
RoadExp:Non-Aligned	0.037	0.018*	−0.002	0.021
	(0.088)	(0.011)	(0.011)	(0.018)
Constant	0.776**	1.234***	0.782***	1.437***
	(0.323)	(0.152)	(0.154)	(0.218)
Observations	31	187	77	110
R^2	0.246	0.156	0.069	0.208
Adjusted R^2	0.017	0.123	−0.026	0.153
Residual Std. Error	0.452 (df = 23)	0.605 (df = 179)	0.354 (df = 69)	0.703 (df = 102)
F Statistic	1.073 (df = 7; 23)	4.720*** (df = 7; 179)	0.726 (df = 7; 69)	3.816*** (df = 7; 102)
Note:				*p<0.1; **p<0.05; ***p<0.01

Model 1 is the Kenyatta era, 1974-78. Model 2 is Moi's full presidency, from 1979-2001. Model 3 is Moi's autocratic era, 1979-1988. Model 4 is Moi's democratic era, 1989-2001.

Electoral returns are available at the Province level. Kenya has seven provinces, whose ethnicity is coded based on whether a majority of their districts are a majority of one ethnic group. Central is coded as Kikuyu, Nyanza is coded as Luo, Rift Valley is coded as Kalenjin, while Coast, Eastern, North-Eastern, and Western are non-aligned.

Figure 5: Schools per 1000 and Moi's Vote Share at Province Level

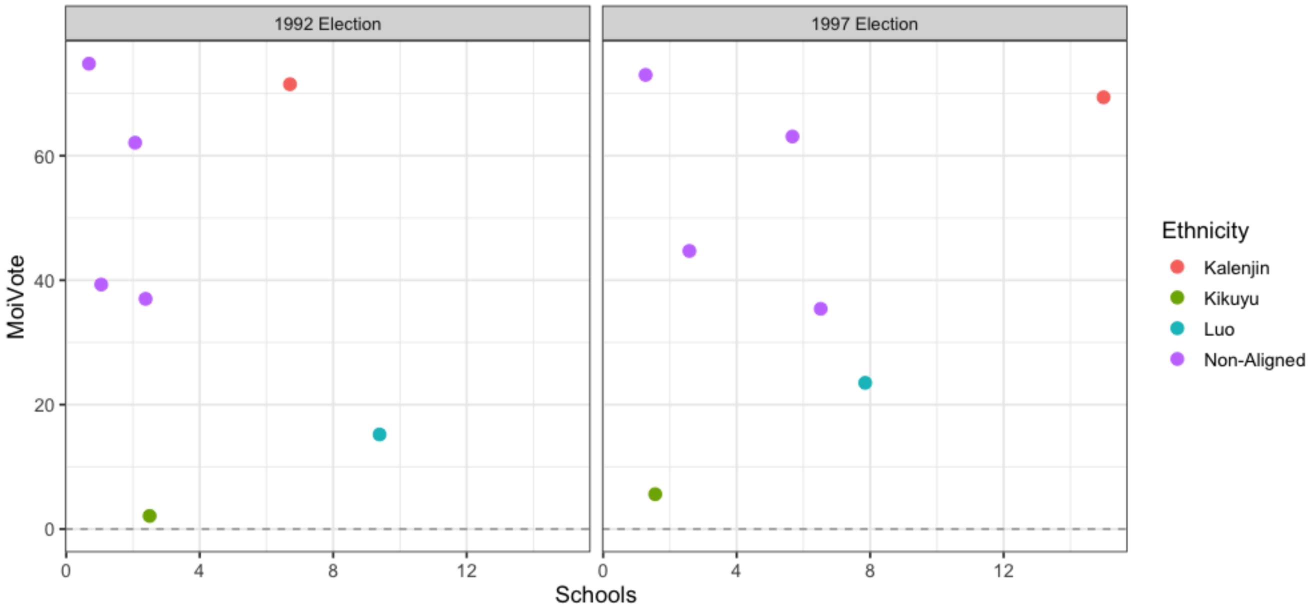
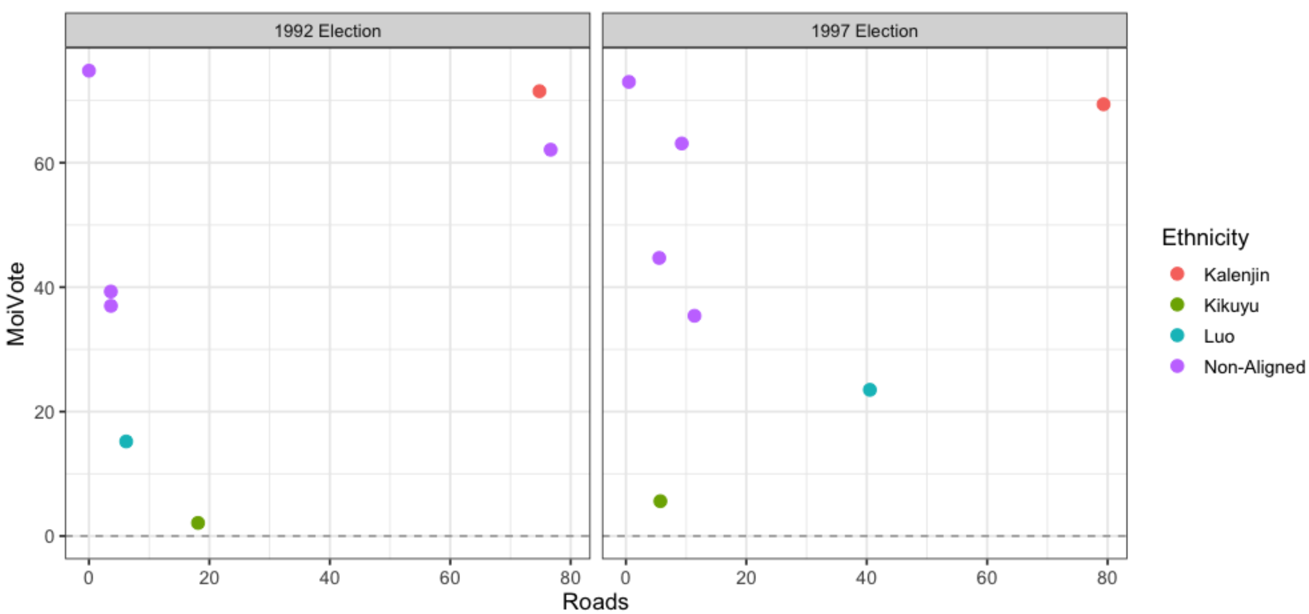


Figure 6: Roads and Moi's Vote Share at Province Level



Formal Appendix

Micro Probabilistic Voting Model

The dictator (D) faces a challenger (C) in an election in some district J. The district has a continuum of voters with unit mass. The winner of district i receives benefit R which I normalize to 1.

Representative voter m_i cares only about their idiosyncratic preference for the dictator relative to the challenger. The utility of voter m_i is

$$U_{m_i} = \begin{cases} V_i & \text{Vote D} \\ 0 & \text{Vote C} \end{cases}$$

V_i denotes the valence of the dictator relative to the challenger for voter m in district i; it is distributed uniformly on $[-\beta_i + \alpha_i, \beta_i + \alpha_i]$ where $\beta_i > 0$ and is uncorrelated with a voter's ideology. While β_i determines the dispersion of V_i, the mean of the distribution is determined by α_i which can be positive or negative. This is natural valence the district has for the dictator prior to any investment. If $\alpha_i > 0$, the dictator is idiosyncratically more popular than the challenger in district i; if $\alpha_i < 0$, the dictator is at a disadvantage; if $\alpha_i = 0$ the district is neutral.

Voter m will vote for the dictator D iff $V_i \geq 0$

If the voter has a positive valence for the dictator, she will vote for the dictator. The probability that $V_i \geq 0$ is $1 - F_{V_i}(0)$. Using the uniform, the probability the valence is sufficient for the dictator to win the election is $\frac{\alpha_i+\beta_i}{2\beta_i}$

Note that $\lim\limits_{\beta_i \to \infty} \dfrac{\alpha_i + \beta_i}{2\beta_i} = \dfrac{1}{2}$ as valence becomes more uncertain, the dictator's expectation of winning goes to $\frac{1}{2}$ regardless of the positive idiosyncratic lean of the district, α_i. In general, the *ex ante* probability that the dictator wins the district is increasing in $\alpha_i > 0$ as he is more popular.

If the dictator invests in mass redistribution in district i, he moves his valence distribution by $\nu_i > 0$. Specifically, $V_i \sim U[-\beta_i + \alpha_i + \nu_i, \beta_i + \alpha_i + \nu_i]$ where $\beta_i, \nu_i > 0$. The probability that the dictator wins is still $1 - F_{V_i}(0)$ which using the uniform is $\frac{\beta_i+\alpha_i+\nu_i}{2\beta_i}$.

General Bargaining SPNE

Denote the dictator's outside option $\Omega \in (0, 1)$. Regardless of the history, after any rejection bargaining continues with common probability $\delta \in (0, 1)$. The size of the regime benefit over which the two parties are bargaining is 1, but note this benefit can be scaled by any constant. The elite makes the first offer.

Let m_E and M_E be the infimum and supremum of equilibrium payoffs to the elite in the game. Let m_D and M_D be the infimum and supremum of equilibrium payoffs to the dictator when he is the proposer. The following inequalities hold:

1. $m_E \geq 1 - (\delta M_D + (1 - \delta)\Omega)$

2. $1 - M_E \geq (\delta m_D + (1 - \delta)\Omega)$

3. $m_D \geq 1 - \delta M_E$

4. $1 - M_D \geq \delta m_E$

In equilibrium, the dictator must accept an offer x where $x = (\delta M_D + (1 - \delta)\Omega)$ as that is the most that he could get from refusing (inequality 1). It follows that the elite cannot get less than w where $w = 1 - (\delta M_D + (1 - \delta)\Omega)$ because she can get a guaranteed w by making it her opening demand.

Similarly, in equilibrium, the dictator must get at least y for each $y = (\delta m_D + (1 - \delta)\Omega)$ because y is guaranteed if the dictator rejects the elite's opening proposal, so the elite can get at most $1 - y$ (inequality 2).

When the dictator is the proposer, the elite must accept an offer x' where $x' = \delta M_E$, the most she could get from refusing (note the elite's outside option is 0). Thus the dictator cannot get less than $1 - \delta M_E$, which he is guaranteed if he makes x' his opening proposal (inequality 3)

The elite must get at least y' for each $y' = \delta m_E$ as that is guaranteed if the elite rejects the dictator's proposal. Thus the dictator can get at most $1 - y'$ (inequality 4).

Rearranging these inequalities, we see that

$M_E \leq 1 - \delta m_D - (1-\delta)\Omega$

$m_E \geq 1 - \delta M_D - (1-\delta)\Omega$

so if $m_D = M_D$, then $m_E = M_E$

Further,

$M_D \leq 1 - \delta m_E$

$m_D \geq 1 - \delta M_E$

So if $m_E = M_E$ then $m_D = M_D$

but how do we know that this is necessarily the case?

Proof by contradiction (to show that $m_D = M_D$)

Assume $m_D < M_D$

From above, we know that $M_D \leq 1 - \delta m_E$ and $m_D \geq 1 - \delta M_E$

$m_D - M_D \geq 1 - \delta M_E - 1 + \delta m_E$ subtracting the lesser from the greater maintains the inequality

$m_D - M_D \geq \delta(m_E - M_E)$

Further,

$M_E \leq 1 - \delta m_D - (1-\delta)\Omega$ and $m_E \geq 1 - \delta M_D - (1-\delta)\Omega$

Thus $m_E - M_E \geq 1 - \delta M_D - (1-\delta)\Omega - (1 - \delta m_D - (1-\delta)\Omega)$ to maintain the inequality

$m_E - M_E \geq \delta(m_D - M_D)$

$\frac{m_E - M_E}{\delta} \geq m_D - M_D$

Combining the above,

$\frac{m_E - M_E}{\delta} \geq m_D - M_D \geq \delta(m_E - M_E)$

By hypothesis, $m_D - M_D < 0$ and as $\delta \in (0,1)$ by definition, $m_E - M_E < 0$

$\frac{m_E - M_E}{\delta} \geq \delta(m_E - M_E)$

$m_E - M_E \geq \delta^2(m_E - M_E)$

This is a contradiction as $\delta \in (0,1)$ and $m_E - M_E < 0$ (multiplying $m_E - M_E$ by a positive number less than one will make it less negative and therefore larger)

Therefore it must be that $m_D \geq M_D$

t the infimum cannot be greater than the supremum by definition

If $m_D \not< M_D$ and $m_D \not> M_D$, is must be that $m_D = M_D$ and, from above, this implies that $m_E = M_E$

Therefore $m_D = M_D$ and $m_E = M_E$. The subgame perfect equilibrium must be unique.

One District Bargaining

Lemma 1. *When the dictator must win one of one districts, for any investment $\nu \geq 0$, the local elite makes him an offer of $\Omega_{1,1} + \frac{\delta}{1+\delta}(1 - \Omega_{1,1})$, which he accepts. If he were to make a counteroffer, the dictator would offer the local elite $\frac{\delta(1+\gamma_1)}{(1+\delta)(1-\gamma_1)}(1 - \Omega_{1,1})$.*

Denote the dictator's outside option as Ω. In the single district case, the dictator's outside option is the probability he wins district i. $\Omega_1 = \frac{\alpha+\beta}{2\beta}$

Stationary strategies: The elite proposes x to the dictator, keeping $1-x$ for herself, every period and accepts the dictator's proposal if and only if $y \geq y'$. The dictator proposes y to the elite, keeping $1-y$ for himself, every period and accepts the elite's proposal if and only if $x \geq x'$

Continuation Values (no investment):

$U_D(Accept|x \geq x') = x'$

$U_D(Reject|x \geq x') = \delta(1 - y') + (1 - \delta)(\Omega_1)$

$U_E(Accept|y \geq y') = y'$

$U_E(Reject|y \geq y') = \delta(1 - x') + (1 - \delta)0$

Set x such that the dictator is indifferent between accepting and rejecting.

$x' = \delta(1 - y') + (1 - \delta)(\Omega_1)$

Similarly set y' such that the elite is indifferent between accepting and rejecting.

$y' = \delta(1 - x') + (1 - \delta)0$

Plug and solve

$x^* = \frac{\Omega+\delta}{1+\delta} = \frac{\alpha+\beta+2\beta\delta}{2\beta(1+\delta)}$

$y^* = \frac{\delta(1+\Omega)}{1+\delta} = \frac{\delta(\beta-\alpha)}{2\beta(1+\delta)}$

Utilities:

$U_E(x^*) = 1 - \frac{\Omega+\delta}{1+\delta} = \frac{\beta-\alpha}{2\beta(1+\delta)}$

$U_D(\neg Invest, Accept\ x^*) = \frac{\Omega+\delta}{1+\delta} = \frac{\alpha+\beta+2\beta\delta}{2\beta(1+\delta)}$

Algebraically adjust to outside option + share of surplus:

$U_D(\neg Invest, Accept\ x^*) = \frac{\Omega+\delta}{1+\delta} = \Omega + \frac{1}{1+\delta}(\Omega + \delta - \Omega(1 + \delta))$

$U_D(\neg Invest, Accept\ x^*) = \Omega + \frac{\delta}{1+\delta}(1 - \Omega)$

Continuation Values (with investment):

$U_D(Accept|x \geq x') = x' - c(\nu)$

$U_D(Reject|x \geq x') = \delta(1 - y') + (1 - \delta)(\Omega_I) = \delta(1 - y') + (1 - \delta)(\frac{\alpha+\beta+\nu}{2\beta}) - c(\nu)$

$U_E(Accept|y \geq y') = y'$

$U_E(Reject|y \geq y') = \delta(1 - x') + (1 - \delta)0$

$x_I^* = \frac{\Omega_I+\delta}{1+\delta} = \frac{\frac{\alpha+\beta+\nu}{2\beta}+\delta}{1+\delta} = \frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)}$

$y_I^* = \frac{\delta(1+\Omega_I)}{1+\delta} = \frac{\delta(\beta-\alpha-\nu)}{2\beta(1+\delta)}$

Utilities:

$U_E(x^*) = \frac{\beta-\alpha-\nu}{2\beta(1+\delta)}$

$U_D(Invest, Accept\ x^*) = \frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)} - c(\nu)$

$U_D(Invest, Accept\ x^*) = \Omega_I + \frac{\delta}{1+\delta}(1-\Omega_I) - c(\nu)$

Note investment solely affects the dictator's outside option of the mass election.

Comparative Statics

The dictator's share is $\frac{\alpha+\beta+2\beta\delta}{2\beta(1+\delta)}$

This is increasing in α, the change in β depends on the sign of α. If α is positive, decreasing in β. If α is negative, increasing in β

The elite's share is $\frac{\beta-\alpha}{2\beta(1+\delta)}$

This is decreasing in α, the change in β depends on the sign of α. If α is positive, increasing in β. If α is negative, decreasing in β

Investment and Surplus

Proposition 8. *When the dictator must win one of one districts and anticipates the equilibrium elite offer of* $\Omega_{1,1} + \frac{\delta}{1+\delta}(1-\Omega_{1,1})$*, he makes optimal investment* $\nu^*_{1,1} = c'^{-1}(\frac{1}{2\beta(1+\delta)})$

While I have focused on the continuous probability of winning the district, the Uniform assumption requires the following cases:

$$F_V(0) = \begin{cases} 0 & \beta < \alpha + \nu \\ \frac{\beta-\alpha-\nu}{2\beta} & -\beta+\alpha+\nu \leq 0 \leq \beta+\alpha+\nu \\ 1 & 0 > \beta+\alpha+\nu \end{cases}$$

Therefore

$$P(\text{win one district}) = \begin{cases} 1 & \beta < \alpha + \nu \\ \frac{\beta+\alpha+\nu}{2\beta} & -\beta+\alpha+\nu \leq 0 \leq \beta+\alpha+\nu \\ 0 & 0 > \beta+\alpha+\nu \end{cases}$$

If $\beta < \alpha + \nu$, the dictator will win the mass election with certainty and the elite will make the only acceptable offer of $x^* = 1$. The dictator's investment problem would then be $\max_\nu 1 - c(\nu)$ and the dictator's optimal investment is 0: there is no benefit to investing as it is impossible to improve his outside option above 0, so the dictator will make no investment.

If $-\beta + \alpha + \nu \leq 0 \leq \beta + \alpha + \nu$

$\max_\nu \frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)} - c(\nu)$

confirm concavity: $\frac{\partial^2}{\partial \nu^2} \frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)} - c(\nu) = 0 - c''$ where $c'' > 0$ by assumption. Thus the objective function is concave.

FOC: $\frac{1}{2\beta(1+\delta)} - c'(\nu) = 0$ set ν such that this holds for optimal investment

Denote the optimal investment when the dictator must win one of one districts to achieve the regime benefit ν_1^*

$\nu_1^* = c'^{-1}(\frac{1}{2\beta(1+\delta)})$

In this internal case, it could be that a large enough investment will move the dictator into a certain victory in the district i.e. there is a $\overline{\nu} < \nu^*$ such that $-\beta + \alpha < 0 < -\beta + \alpha + \overline{\nu}$

If the dictator makes investment $\underline{\nu}$, his expected offer from the elite is 1 and his expected overall utility will be $1 - c(\overline{\nu})$ where $1 - c(\overline{\nu}) > \frac{\alpha+\beta+2\beta\delta+\nu^*}{2\beta(1+\delta)} - c(\nu^*)$ as $c(\overline{\nu}) < c(\nu^*)$ by definition and $\frac{\alpha+\beta+2\beta\delta+\nu^*}{2\beta(1+\delta)} < 1$. Thus the dictator will make investment $\overline{\nu}$ to maximize his outside option to an outside win probability of 1 and keep the full regime benefit less the cost of investment.

If $\beta + \alpha + \nu < 0$, the dictator will lose the mass election. It could be the case that sufficient investment will move him from a win probability of 0 to a positive district win probability. Let $\underline{\nu}$ be the minimal level of investment the dictator needs to make such that $\beta + \alpha + \underline{\nu} \geq 0 > \beta + \alpha$. Consider cases:

(1) The dictator's optimal investment ν^* is greater than $\underline{\nu}$.

The dictator maximizes $\max_\nu \frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)} - c(\nu)$ $s.t.$ $\nu \geq \underline{\nu}$

As $\nu^* = c'^{-1}(\frac{1}{2\beta(1+\delta)}) \geq \underline{\nu}$, the constraint is satisfied and the dictator will make his optimal investment and increase his probability of winning the district alone from 0 to a positive probability.

(2) It could be the case that $\nu^* < \underline{\nu}$. If the dictator makes investment $\underline{\nu}$, he will increase his probability of winning from 0 to something minimally positive, but at cost $c(\underline{\nu})$

If he does not make the investment, his outside option is 0, so he expects the elite will offer him $\frac{\delta}{1+\delta}$, which he will accept. If he invests, $\underline{\nu}$, he expects the elite will offer him$\frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)} - c(\underline{\nu})$. Should he invest?

$\frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)} - c(\underline{\nu}) - \frac{\delta}{1+\delta} \geq 0$

$\frac{\alpha+\beta+\nu}{2\beta+2\beta\delta} - c(\underline{\nu}) \geq 0$

$\frac{\alpha+\beta+\nu}{2\beta+2\beta\delta} \geq c(\underline{\nu})$

$\frac{\alpha+\beta+\nu}{2\beta} \geq c(\underline{\nu})(1+\delta)$

The boost in his outside option that the dictator receives from investing a sufficient amount to generate a positive win probability in the district must outweigh the cost of investment (scaled).

Investment with convex cost functional form: $c(\nu) = \frac{\nu^2}{2}$

$U_D = \frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)} - c(\nu)$

$U_D = \frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)} - \frac{\nu^2}{2}$

$\max_\nu \frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)} - \frac{\nu^2}{2}$

$\nu^* = \frac{1}{2\beta(1+\delta)}$

Decreasing in β, decreasing in δ

Surplus

Mutual surplus $\equiv E[U_D(intermediary)] + E[U_E(intermediary)] - (E[U_D(\neg intermediary)] + E[U_E(\neg intermediary)])$

$= 1 - (\Omega + 0)$

The total benefit of using the elite intermediary is 1 which will be divided according to the equilibrium bargain. The total benefit if the dictator does not use the elite is $\frac{\alpha+\beta}{2\beta}$. We could describe this as being divided 1, 0 between the dictator and elite, respectively. So the mutual surplus of going through the elite is $1 - \frac{\alpha+\beta}{2\beta} = \frac{\beta-\alpha}{2\beta}$. This mutual surplus is decreasing in α (if the dictator is ex ante popular, going without the elite approaches the using the elite as α increases). How the mutual surplus changes in β depends on α. If α is positive, the mutual surplus is increasing in β. If α is negative, the mutual surplus is decreasing in β.

Without investment ($\nu = 0$), the change in the dictator's share in β depends on the dictator's *ex ante* popularity: if the dictator is initially popular such that $\alpha > 0$, the dictator's share is decreasing in β. Increased uncertainty counteracts the dictator's popularity, reducing his outside option and worsening his bargaining position. If the dictator is *ex ante* unpopular, however, his share is increasing in β. Increasing uncertainty makes an unpopular dictator more likely to win, increasing his outside option and bargaining position.

With investment, the optimal investment is in itself a function of β

Share with investment: $\frac{\alpha+\beta+2\beta\delta+\nu^*(\beta)}{2\beta(1+\delta)} = \frac{\alpha+\beta+2\beta\delta}{2\beta(1+\delta)} + \frac{\nu^*(\beta)}{2\beta(1+\delta)}$

$\frac{\partial}{\partial\beta}\frac{\alpha+\beta+2\beta\delta}{2\beta(1+\delta)} + \frac{\nu^*(\beta)}{2\beta(1+\delta)} = -\frac{\alpha}{2\beta^2(1+\delta)} + \nu^{*'}(\beta)(\frac{1}{2\beta(1+\delta)}) + -\frac{1}{2\beta^2(1+\delta)}\nu^*(\beta)$

I know that $v^* = c^{'-1}(\frac{1}{2\beta(1+\delta)})$ is decreasing in β so $\nu^{*'}$ is negative (and ν^* is positive unless it is 0).

So if α is positive, $\frac{\partial share}{\partial\beta}$ is definitely negative. But if α is negative, the magnitudes determine the sign which would require a functional form for c to determine.

$\frac{-(\alpha+\nu^*(\beta))}{2\beta^2(1+\delta)} + \frac{\nu^{*'}(\beta)}{2\beta(1+\delta)}$ the second term is negative, the sign of the first term depends on the relative magnitude of α and $\nu^*(\beta)$

Investment Comparison Without Bargaining Technology

Let ν_E^* denotes the dictator's optimal investment when he uses an elite intermediary and ν_M^* is the dictator's optimal investment when he goes directly to the masses without elite cooptation.

Elite Bargain Objective Function: $\max_\nu \frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)} - c(\nu)$

No Elite Intermediary Objective Function: $\max_\nu \frac{\alpha+\beta+\nu}{2\beta} - c(\nu)$

As c' is invertible, $\nu_E^* = c^{'-1}(\frac{1}{2\beta(1+\delta)})$ and $\nu_M^* = c^{'-1}(\frac{1}{2\beta})$. Note that $\nu_M^* > \nu_E^*$: the dictator's optimal level of investment if there is not elite intermediary is greater than the optimal level of investment with the elite intermediary.

Extreme Bargaining Protocols

Dictator makes the elite a take-it-or-leave-it offer (dictator extremely advantaged)

Sequence of play:

The dictator makes an investment decision of $\nu \in [0, 1]$ at convex cost $c(\nu)$

The dictator makes a TIOLI offer of e to the elite

The elite accepts the offer, receiving e, or rejects, leaving her with her reservation utility of 0

If the dictator coopted the elite, he wins the election with certainty; without the elite he runs for election in the district unsupported according to the micro-voting model (win with probability $\frac{\alpha+\beta+\nu}{2\beta}$. If the dictator wins the district, he keeps regime benefit $R = 1$ less any offer he made to the elite

The dictator will make the elite an offer of $e = 0$ as the elite will be indifferent between accepting and rejecting the offer (accept on indifference). As the dictator will win the district with certainty without having to transfer anything to the elite, his investment optimization problem is $\max_\nu 1 - c(\nu)$ and will thus make 0 investment in mass politics.

Elite makes the dictator a take-it-or-leave-it offer (elite extremely advantaged)

Sequence of play:

The dictator makes an investment decision of $\nu \in [0,1]$ at convex cost $c(\nu)$

The elite makes the dictator a TIOLI offer of d, if the dictator accepts her offer, she will keep $1-d$

The dictator accepts the offer, receiving d with certainty, or rejects and stands for the election unsupported

If the dictator coopted the elite, he wins the district with certainty; without the elite he runs for election in the district unsupported according to the micro-voting model (win with probability $\frac{\alpha+\beta+\nu}{2\beta}$. If the dictator wins the district, he keeps regime benefit $R = 1$ less any deal made with the elite

The dictator's expected utility is $d - c(\nu)$ if he accepts the elite's proposal and $\frac{\alpha+\beta+\nu}{2\beta}(1) - c(\nu)$ if he rejects the elite's proposal. To maximize her utility, the elite will make the dictator a minimal offer that makes her indifferent between accepting and rejecting: $d = \frac{\alpha+\beta+\nu}{2\beta}$, which the dictator accepts. The dictator's investment optimization problem is thus $\max_\nu \frac{\alpha+\beta+\nu}{2\beta} - c(\nu)$. The dictator's optimal investment here is $\nu^* = c'^{-1}(\frac{1}{2\beta})$, which is equivalent to ν^*_M

Under more extreme bargaining protocols, the dictator's optimal investment is less than or equal to the investment he makes without the elite bargaining technology. In particular, if the dictator is in the extreme privileged bargaining position and can make a take-it-or-leave it offer to the local elite, the elite will support him with an offer of 0 as she will accept on indifference. Thus the dictator will make an investment of 0, give the elite 0, and still win the regime benefit with certainty. In the other extreme, if the elite can make a take-it-or-leave-it offer to the dictator, the minimal offer dictator will accept is that which makes him indifferent between using the elite's support or standing for election alone ($\frac{\alpha+\beta+\nu}{2\beta}$). In this case, the dictator's optimal investment is $c'^{-1}(\frac{1}{2\beta})$, which is the same as ν_M^*. Thus the optimal investment the dictator will make under the Rubinstein alternating offer bargaining protocol is between the optimal investments he would make if he were strongly advantaged (0) or disadvantaged (ν_M^*) by the bargaining protocol.

Two Homogeneous Districts

Two Districts: Two of Two

First consider a two-district state where the dictator needs the support of both districts in order to achieve the regime benefit. The dictator still has the option of investing in mass politics, which increases his likelihood of mass election uniformly in all districts. As described above, nature chooses which elite the dictator bargains with first and, after that bargain is concluded either with an agreement or breakdown, the dictator can then bargain with the second elite before the election occurs. Without bargaining, the likelihood that the dictator wins both districts through the election, and thus his expected utility for the no-elite no-investment world, is $\Omega_{2,2} = (\Omega_{1,1})^2 = \gamma^2$. As above, this is increasing in his popularity among voters and decreasing in electoral uncertainty if he is popular, increasing in electoral uncertainty if he is unpopular.

If the dictator and the elite in the first bargaining position fail to reach an agreement, the dictator's bargain with the elite in the second position is scaled by $\Omega_{1,1}$, the probability he wins one district alone. This is because even if they reach an agreement, the dictator will only get the regime benefit to split with the local elite if he wins the other district, which occurs with probability $\Omega_{1,1}$. The maximum mutual surplus that the dictator and second position elite are bargaining over is $1 * \Omega_{1,1}$. If the dictator and second elite also fail to reach an agreement, the dictator's true electoral outside option is $(\Omega_{1,1})^2$, the probability he wins both districts alone. Thus the bargain that the dictator and second elite will come to is

$(\Omega_{1,1})^2 + \dot{\lambda}(\Omega_{1,1} - (\Omega_{1,1})^2) = \Omega_{1,1}(\Omega_{1,1} + \dot{\lambda}(1 - \Omega_{1,1}))$. As we know the dictator will make a deal with the first elite, this is off path.

If the dictator and first elite do reach an agreement, the dictator's bargain with the second elite is scaled by x_1, the amount of the regime benefit the dictator is left with after the agreement with the first elite. The maximum mutual surplus over which the two parties bargain is x_1. If the dictator and second elite fail to reach an agreement in this case, the outside option is what the dictator gets if he and the second elite fail to reach an agreement: $\Omega_{1,1}x_1$. Thus the bargain they come to is $\Omega_{1,1}x_1 + \lambda(x_1 - \Omega_{1,1}x_1) = x_1(\Omega_{1,1} + \lambda(1 - \Omega_{1,1}))$. Therefore in equilibrium we know that the dictator will keep $(\Omega_{1,1} + \lambda(1 - \Omega_{1,1}))$ portion of whatever is left after his bargain with the first elite. The first elite will leave him with $(\Omega_{1,1} + \lambda(1 - \Omega_{1,1}))$ between the first and second round of bargaining, so the dictator's utility at the end of both bargains after coopting both districts is $(\Omega_{1,1} + \lambda(1 - \Omega_{1,1}))^2$.

Lemma 12. *When the dictator must win two of two districts, for any investment $\nu \geq 0$, each homogeneous elite makes him an offer of $\Omega_{1,1} + \lambda_{1,1}(1 - \Omega_{1,1})$, which he accepts. If he were to make a counteroffer, the dictator would offer each elite $\frac{\delta(1+\gamma)}{(1+\delta)(1-\gamma)}(1 - \Omega_{1,1})$.*

Note that the individual bargains that the dictator conducts with each elite precisely follow his bargains in the one of one institutional arrangement. As he needs both elites and successfully coopting only one does not guarantee him the regime benefit (though it does affect his probability of winning it), each individual bargain is the same despite the difference in his general outside option. Both elites offer $\Omega_{1,1} + \lambda_{1,1}(1 - \Omega_{1,1})$, so the dictator's utility is $(\Omega_{1,1} + \lambda_{1,1}(1 - \Omega_{1,1}))^2 - c(\nu)$. Using his true outside option in this institutional configuration, the dictator's utility is $\Omega_{2,2} + \lambda_{2,2}(1 - \Omega_{2,2}) - c(\nu)$ where $\lambda_{2,2} \equiv \frac{\delta^2}{(1+\delta)^2} + \frac{2\delta\gamma}{(1+\delta)^2(1+\gamma)}$ (which is equivalent to $(\lambda_{1,1})^2 + \frac{2\delta\gamma}{(1+\delta)^2(1+\gamma)}$). As the dictator's outside option (winning with an election only) is lower when he has to win two districts instead of one, the surplus that he shares with the elite is greater. The share of the surplus that the dictator keeps, $\lambda_{2,2}$, is less than his single district share $\lambda_{1,1}$ as the elite are in a strong bargaining position with the surplus that they offer and the dictator is splitting the regime benefit with not just one but both elites.[18]

Prior to bargaining with the elites, the dictator can make his investment decision. For now, I assume that a single investment ν at cost $c(\nu) = \frac{\nu^2}{2}$ affects all districts uniformly: the dictator's popularity increases by ν in both districts.

Proposition 8. *When the dictator must two of two districts, anticipates the equilibrium elite offers of* $(\Omega_{1,1} + \lambda_{1,1}(1 - \Omega_{1,1}), \Omega_{1,1} + \lambda_{1,1}(1 - \Omega_{1,1}))$*, he makes optimal investment which satisfies the first order condition* $\frac{\alpha+\beta+2\beta\delta+\nu}{2\beta^2(1+\delta)^2} - c'(\nu) = 0$.

Corollary 3. *When the dictator must win two of two districts, anticipates the equilibrium elite offers of* $(\Omega_{1,1} + \lambda_{1,1}(1 - \Omega_{1,1}), \Omega_{1,1} + \lambda_{1,1}(1 - \Omega_{1,1}))$*, and* $c(\nu) = \frac{\nu^2}{2}$*, he makes optimal investment*

$$\nu^*_{2,2} = \begin{cases} 0 & \beta \leq \sqrt{\frac{1}{2}} \\ \frac{\alpha+\beta+2\beta\delta}{2\beta^2(1+\delta)^2-1} & \beta > \sqrt{\frac{1}{2}} \end{cases}$$

When the uncertainty of the election gets extremely low (β is extremely low), the dictator's utility gets infinitely large. Any investment in this region of the parameter space does materially change his utility, he only incurs the cost of putting resources into mass distribution. Thus only when there is some uncertainty in his electoral outside option will the dictator invest in his popularity among citizens. Similar to the single district state, optimal investment is increasing in the dictator's popularity and decreasing in electoral uncertainty when the dictator is popular. If he is sufficiently unpopular, investment is increasing in uncertainty.[19]

[18]An **individual** elite's portion of the surplus is larger in the single district setting than two of two ($(1 - \lambda_{1,1}) > \frac{(1-\lambda_{2,2})}{2}$).

[19]Specifically if $\alpha < -\frac{1+2\beta^2}{4\beta}$

Two Districts: One of Two

Consider a state with two districts of which the dictator only needs one to succeed. Even without the elite intermediaries, the dictator's baseline outside option of winning the election is now $\Omega_{1,2} = 2\gamma - \gamma^2$.[20] As above, the dictator's likelihood of winning sufficient districts is increasing in his popularity and decreasing in the uncertainty of the election if he is popular. Note that without the elites, the dictator is unambiguously better off when he needs to only win one of two instead of two of two districts as his likelihood of winning sufficient districts is higher. In addition to the increase in his electoral outside option, only needing one of two elites supports the dictator's bargaining position. Because he only needs one district, if the dictator achieves an agreement with the first elite, the second is completely superfluous: no offer of sharing will entice the dictator to accept. The second elite will be left with nothing and the dictator will only share the regime benefit with the first elite. Despite her exclusion from the regime benefit, the second elite still matters a great deal: the bargain that the dictator strikes with the elite in the first bargaining position is materially different from the one-district bargain.

Lemma 13. *In equilibrium, the elite in the second bargaining position will never receive a portion of the regime benefit greater than 0.*

Lemma 14. *When the dictator must win one of two districts, for any investment $\nu \geq 0$, the local elite in the first bargaining position makes him an offer of $\Omega_{1,2} + \lambda_{1,2}(1 - \Omega_{1,2})$, which he accepts; if he were to make a counteroffer, the dictator would offer the first elite $\frac{\delta+\delta\gamma(2-\gamma)}{(1+\delta)(\gamma-1)^2}(1 - \Omega_{1,2})$. The local elite in the second bargaining position makes the dictator an offer of 1, which he accepts; if he were to make a counteroffer, the dictator would offer the second elite 0.*

As discussed above, having a second district to win but only needing one changes the probability that the dictator is successful in a mass election, his outside electoral option

[20]Recall γ is the generic probability the dictator wins a single district. With the micro-voting model, $\gamma = \frac{\alpha+\beta}{2\beta}$.

$\Omega_{1,2}$. The probability of winning one of two districts is always greater than or equal to the probability of winning one of one or one of two.[21] While the dictator's electoral outside option is stronger, this also reduces the surplus from using the elite intermediaries. Further, having an additional potential bargaining partner increases the share of the regime benefit that the dictator can keep. If the dictator rejects the first elite's offer and their negotiations breakdown, he still has the opportunity to bargain with the second elite rather than being forced into an immediate election. Thus the elite has to give up more of the mutual surplus $(1 - \Omega_{1,2})$ in order to get the dictator to agree to his terms; if he were to demand as much as the local elite in the one-district or two of two version, the dictator would simply move on to the second elite and leave the first with nothing. Note that the elites (the districts they represent) are homogeneous: the only difference between them is the nature-chosen bargaining position that determines whether they share in the regime benefit or are left with nothing.

This change in institutional environment–where the dictator now has a back-up bargaining partner–can be seen in the difference between $\lambda_{1,1}$, the portion of the mutual surplus the dictator got with one district, and $\lambda_{1,2}$ (which is $\frac{\delta(2+\delta)}{(1+\delta)^2}$), the portion of the surplus he gets with needing only one of two. Note that $\lambda_{1,2} > \lambda_{1,1}$ for all $\delta > 0$: if there is any opportunity to bargain, the dictator keeps more of the mutual surplus when he has a second, superfluous district.[22] Having a superfluous district, while beneficial for the dictator's overall utility, complicates his investment decision.

Proposition 9. *When the dictator must win one of two districts and anticipates the equilibrium elite offers of* $(\Omega_{1,2} + \frac{\delta(2+\delta)}{(1+\delta)^2}(1 - \Omega_{1,2}), 1)$*, he makes optimal investment which satisfies the first order condition* $\frac{\beta-\alpha-\nu}{2\beta^2(1+\delta)^2} - c'(\nu) = 0$*.*

[21] Assuming, as I do here, that the individual district probabilities are independent and the same.
[22] $\lambda'_{1,2} > \lambda_{2,2}$ as well

Corollary 4. *When the dictator must win one of two districts, anticipates the equilibrium elite offers of* $(\Omega_{1,2}+\frac{\delta(2+\delta)}{(1+\delta)^2}(1-\Omega_{1,2}),1)$ *and his convex cost function* $c(\nu)$ *takes the functional form* $\frac{\nu^2}{2}$*, he makes optimal investment* $\nu^*_{1,2}=\frac{\beta-\alpha}{1+\beta^2(1+\delta)^2}$.

Utilizing this simple functional form for costs, the dictator's optimal investment balances the increase in his electoral outside option with the negative effect that mass investment has on the mutual surplus as well as its outright cost. The optimal investment in this case is decreasing in α, the dictator's *ex ante* popularity. The dictator is strongly advantaged by both having a superfluous back-up bargaining partner as well as the increased outside option simply from needing one of two districts; additional investment in his outside option via mass redistribution is less helpful. The dictator's equilibrium investment further depends on the uncertainty of the election: if he is sufficiently popular,[23] investment is increasing in the uncertainty of the electoral environment. At the upper end of his utility function (high α) the marginal returns to increasing his popularity are extremely small (flat slope), so an increase in β extends the domain and makes investment more lucrative. In other words, when the dictator is extremely popular he will already invest very little, but an increase in the uncertainty of the electoral environment gives him room to invest more in his popularity.

While the movement from needing one of one to two of two changed the dictator's outside option and share of the regime benefit he kept, it did not materially change his incentives to invest in mass politics. In this institutional set up, however, the dictator no longer uses investment to simply increase his popularity (investing more when he is already popular). Instead, the dictator invests more when he is *unpopular*: investment is decreasing in his popularity, α. As the one of two institutional arrangement is so beneficial to the dictator in terms of both his outside option and the outcome of his bargain, he does not need to maximize his popularity but instead minimize his unpopularity.

[23] $\alpha>\frac{\beta^2(1+\delta)^2-1}{2\beta(1+\delta)^2}$

The Effect of Institutions: Homogeneous Baseline

Despite using exactly the same bargaining protocol and homogeneous elites, the institutional rules of how many districts the dictator needs to win the regime benefit have drastic consequences for both the amount of the regime benefit the elites can extract and the dictator's optimal investment. Increasing the dictator's electoral outside option–his chances of achieving the regime benefit without local elite support– both increases the baseline demand he can make, but also reduces the surplus that using an elite intermediary generates. Having a superfluous district, however, unambiguously increases the dictator's utility through strengthening his bargaining position. Because he still has an opportunity to coopt the second elite and win the regime benefit, the dictator's position vis-a-vis the first elite he bargains with is much more advantaged. These simultaneous effects of altering the dictator's bargaining position relative to the elite and altering both the electoral outside option and surplus that the elite intermediary technology generates demonstrates the complexity of unit proliferation from the dictator's perspective. Where he to choose his own institutional situation, having to win one of two is preferred. However, general unit proliferation (the creation of more districts) does not necessarily imply that he will still only need a bare majority to win.

The varied institutional configurations also yields different predictions regarding the dictator's optimal investment in mass redistribution to increase his electoral popularity. Recall that the dictator will invest the most in mass redistribution when there are no elite intermediaries to bargain with: if he must stand for election without elite support, he will invest more in mass redistribution as the effect of his investment on his expected utility is direct. Investment with elite intermediaries is still beneficial to the dictator: increases his outside option improves his bargaining position, but because he does not actually stand for election unsupported, the effect is indirect. Which institutional configuration yields the greatest investment in mass politics? Which optimal investment is greater depends on the dictator's *ex ante* popularity, α. Holding the other parameters (β, δ) constant, if the dictator is unpopular such that $\alpha < -\frac{1+2\beta^2\delta(1+\delta)}{2\beta(1+\delta)}$, the dictator's optimal investment under one of two districts is greater than his optimal investment when he must win two of two. If he is not this unpopular ($\alpha > -\frac{1+2\beta^2\delta(1+\delta)}{2\beta(1+\delta)}$), his optimal investment when he needs to win two of two is greater. Recall that the dictator wants to invest more in his popularity when he is already popular when he needs to win two of two districts: his optimal investment is increasing in his popularity as the marginal benefit of increasing his popularity is higher when he is already popular. When he needs to win one of two districts, however, optimal investment highest when the dictator is unpopular: the marginal benefit of increasing his popularity is highest when α is extremely low.

Note that these relationships between the institutional environment, the dictator's electoral outside option, and the dictator's optimal investment are not particular to these particular institutions. When the dictator needs to win two of three districts and maintains that superfluous bargaining partner, his optimal investment mimics his optimal investment under one of two.[24] See the appendix for a full treatment and discussion of the electoral outside option, equilibrium offers, and optimal investment when the dictator needs to win two of three districts.

Two Districts

There are two homogeneous districts. The value of the regime is normalized to 1.

First, consider the case in which the dictator must win at least one district to receive this benefit (need at least one of two).

Nature chooses which elite bargains with the dictator first, then bargaining occurs as described above. After the dictator completes his bargain with the first elite (either through an agreement or breakdown), the dictator bargains with the second elite following the same

[24]In general his optimal investment is decreasing in his popularity except for the extremes of α where the slope of his electoral option is very flat.

protocol. Both the probability of bargaining breakdown and the probability the dictator wins each district is the same for both elites/districts. After the dictator completes his bargain with the second elite (either through an agreement or breakdown), the elections are held in both districts.

$P(wini) = \frac{\alpha+\beta}{2\beta}$ (from micro model)

$P(win\ at\ least\ one) = \Omega_2' = 2(\frac{\alpha+\beta}{2\beta}) - (\frac{\alpha+\beta}{2\beta})^2$ (win one or both districts)

Second Bargaining Position

Consider the subgame in which the first elite has already been coopted. Because the dictator has already made an agreement with the first elite, they are now bargaining over the dictator's remaining portion. Let x_1 be the portion of the benefit that the dictator is left with after the first bargain. The second elite will propose $x_2 x_1$, dividing x_1 into two portions: x_2 for the dictator, $1 - x_2$ for himself. Similarly, the dictator proposes a split of $(1 - y_2)x_1$ for himself and $y_2 x_1$ for the second elite.

This yields continuation values:

$U_D(accept x_2) = x_2 x_1$

$U_D(reject) = \delta(1 - y_2)x_1 + (1 - \delta)x_1$

$x_2 = 1 - \delta y_2$

$U_E(accept y_2) = y_2 x_1$

$U_E(reject) = \delta(1 - x_2)x_1 + (1 - \delta)0$

$y_2 = \delta(1 - x_2)$

plug and solve

$x_2^* = 1$ the dictator gets everything, as expected

$y_2^* = 0$ the elite gets nothing, as expected

Because the dictator has already successfully coopted the first elite, the second district is superfluous. There is no offer of sharing that the dictator would accept.

Consider the subgame in which the first elite has not been coopted.

$U_D(accept) = x_2$

$U_D(reject) = \delta(1-y_2) + (1-\delta)\Omega_2' = \delta(1-y_2) + (1-\delta)(2(\frac{\alpha+\beta}{2\beta}) - (\frac{\alpha+\beta}{2\beta})^2)$ [the probability that the dictator wins one or both districts]

$U_E(accept) = y_2$

$U_E(reject) = \delta(1-x_2) + (1-\delta)(0)$

$x_2^* = \frac{\Omega_2'+\delta}{1+\delta} = \frac{2(\frac{\alpha+\beta}{2\beta})-(\frac{\alpha+\beta}{2\beta})^2+\delta}{1+\delta} = \frac{(\frac{\alpha+\beta}{\beta})-(\frac{\alpha+\beta}{2\beta})^2+\delta}{1+\delta}$

$y_2^* = \frac{\delta(1+\Omega_2')}{1+\delta} = \frac{(\alpha-\beta)^2\delta}{4\beta^2(1+\delta)}$

Utilities: $U_E(x_2^*) = \frac{(\alpha-\beta)^2}{4\beta^2(1+\delta)}$

$U_D(Accept) = \frac{(\frac{\alpha+\beta}{\beta})-(\frac{\alpha+\beta}{2\beta})^2+\delta}{1+\delta}$

First Bargaining Position

If the dictator coopts the first elite with successful bargaining, he gets x_1. If bargaining breaks down and elite 1 is not coopted, the dictator will bargain with the second elite and we know that bargain will yield $\Omega_2' = \frac{\Omega_2'+\delta}{1+\delta} = \frac{(\frac{\alpha+\beta}{\beta})-(\frac{\alpha+\beta}{2\beta})^2+\delta}{1+\delta}$.

$U_D(accept) = x_1$

$U_D(reject) = \delta(1-y_1) + (1-\delta)\Omega_2' = \delta(1-y_1) + (1-\delta)(\frac{(\frac{\alpha+\beta}{\beta})-(\frac{\alpha+\beta}{2\beta})^2+\delta}{1+\delta})$

$U_E(accept) = y_1$

$U_E(reject) = \delta(1-x_1) + (1-\delta)(0)$

$x_1^* = \frac{\Omega_2'+\delta}{1+\delta} = \frac{(\frac{\Omega_2'+\delta}{1+\delta})+\delta}{1+\delta} = \frac{(\alpha+\beta+2\beta\delta)(\beta(3+2\delta)-\alpha)}{4\beta^2(1+\delta)^2}$ [with investment $\frac{(\alpha+\beta+2\beta\delta+\nu)(\beta(3+2\delta)-\alpha-\nu)}{4\beta^2(1+\delta)^2}$]

$y_1^* = \frac{\delta(1+\Omega_2')}{1+\delta} = \frac{\delta(\alpha-\beta)^2}{4\beta^2(1+\delta)^2}$

Utilities: $U_D(acceptx_1^*) = \frac{(\alpha+\beta+2\beta\delta)(\beta(3+2\delta)-\alpha)}{4\beta^2(1+\delta)^2}$

$U_E(x_1^*) = \frac{(\alpha-\beta)^2}{4\beta^2(1+\delta)^2}$

Substitute in Ω'_2:

With two districts (need one of two), $U_D(x_1^*, x_2^*) = \Omega'_2 + \frac{\delta(2+\delta)}{(1+\delta)^2}(1-\Omega'_2)$ where Ω'_2 is the probability the dictator wins one or both districts without using an elite intermediary.

the dictator accepts the elite's first offer, x_1^*. The elite in the second bargaining position is indifferent between making an offer of 1 (where the dictator gets everything and the elite gets nothing) and making unacceptable offers until bargaining breaks down (she gets zero either way). The dictator's overall utility is $\frac{(\alpha+\beta+2\beta\delta)(\beta(3+2\delta)-\alpha)}{4\beta^2(1+\delta)^2}$. For the elites, their utility depends on the bargaining position nature selects for them. Let the probability that an elite is chose for the first position be ρ. Both elites' expected utility is then $E[U_E(x_1^*, x_2^*)] = \rho(\frac{(\alpha-\beta)^2}{4\beta^2(1+\delta)^2}) + (1-\rho)(0)$. As the probability of going first is symmetric for both elites, $\rho = \frac{1}{2}$

Comparative Statics: $\frac{\partial}{\partial\alpha}U_D(acceptx_1^*) = \frac{(\alpha+\beta+2\beta\delta)(\beta(3+2\delta)-\alpha)}{4\beta^2(1+\delta)^2} = \frac{\beta-\alpha}{2\beta^2(1+\delta)^2}$ which is positive. The dictator's share (and equilibrium utility) are increasing in his ex ante popularity.
$\frac{\partial}{\partial\beta}U_D(acceptx_1^*) = \frac{(\alpha+\beta+2\beta\delta)(\beta(3+2\delta)-\alpha)}{4\beta^2(1+\delta)^2} = \frac{\alpha(\alpha-\beta)}{2\beta^3(1+\delta)^2}$ the sign of which depends on α (if $\alpha > 0$, decreasing in β, if $\alpha < 0$ increasing in β)
$\frac{\partial}{\partial\alpha}U_E(x_1^*) = \frac{(\alpha-\beta)^2}{4\beta^2(1+\delta)^2} = \frac{\alpha-\beta}{2\beta^2(1+\delta)^2}$ which is negative, decreasing in the dictator's popularity
$\frac{\partial}{\partial\beta}U_E(x_1^*) = \frac{(\alpha-\beta)^2}{4\beta^2(1+\delta)^2} = -\frac{\alpha(\alpha-\beta)}{2\beta^3(1+\delta)^2}$ the sign of which depends on α.

Mutual Surplus $= 1 - (\frac{\alpha+\beta}{\beta} - (\frac{\alpha+\beta}{2\beta})^2)$ note this is different than the one district mutual surplus because now if the dictator bypasses the elite he has two possible districts of which he must win one.
$1 - (\frac{\alpha+\beta}{\beta} - (\frac{\alpha+\beta}{2\beta})^2) = \frac{(\alpha-\beta)^2}{4\beta^2}$
The mutual surplus is decreasing in α and how it changes in β depends on the sign of α (if positive, increasing in β, if negative decreasing in β)

Compare utilities with one district versus two

Elites: One district elite - two districts elite $= \frac{\beta-\alpha}{2\beta(1+\delta)} - (\rho(\frac{(\alpha-\beta)^2}{4\beta^2(1+\delta)^2}) + (1-\rho)0) = \frac{(\beta-\alpha)(\alpha+\beta(3+4\delta))}{8\beta^2(1+\delta)^2}$

Dictator: Two districts - one district $= \frac{(\alpha+\beta+2\beta\delta)(\beta(3+2\delta)-\alpha)}{4\beta^2(1+\delta)^2} - \frac{\alpha+\beta+2\beta\delta}{2\beta(1+\delta)} = \frac{(\beta-\alpha)(\alpha+\beta+2\beta\delta)}{4\beta^2(1+\delta)^2}$ literally transfers some gain to the dictator recall $\beta > \alpha$ for a proper probability)

Investment

Note the probability of winning one of two districts is non-monotonic in investment. For investment greater than 0, the probability of winning one of two districts is non-monotonic in α. Thus the optimal investment depends on α

Equilibrium investment with two districts:

$\max_{\nu} \frac{(\alpha+\beta+2\beta\delta+\nu)(\beta(3+2\delta)-\alpha-\nu)}{4\beta^2(1+\delta)^2} - c(\nu)$

FOC is $\frac{\beta-\alpha-\nu}{2\beta^2(1+\delta)^2} - c'(\nu) = 0$

Note optimal investment is decreasing in α

Compare to investment when dictator needs one of one districts:

One district FOC: $\frac{1}{2\beta(1+\delta)} - c'(\nu) = 0$

Assume $c(\nu) = \frac{\nu^2}{2}$ such that $c'(\nu) = \nu$

$\nu^*_{2D} = \frac{\beta-\alpha}{1+2\beta^2(1+\delta)^2}$

Comparative Statics:

$\frac{\partial \nu^*_{2D}}{\partial \alpha} = \frac{-1}{1+2\beta^2(1+\delta)^2}$ investment is decreasing in the dictator's popularity

$\frac{\partial \nu^*_{2D}}{\partial \beta} = \frac{1+2(2\alpha-\beta)\beta(1+\delta)^2}{(1+2\beta^2(1+\delta)^2)^2}$

This is not always negative. if α is sufficiently high, this is positive. In particular, if $\alpha > \frac{\beta}{2} - \frac{1}{4\beta(1+\delta)^2}$, the derivative is positive and investment is increasing in district uncertainty.

$\nu^*_{1D} = \frac{1}{2\beta(1+\delta)}$

$\nu^*_{1D} - \nu^*_{2D} = \frac{1+2\alpha\beta(1+\delta)+2\beta^2\delta(1+\delta)}{2\beta(1+\delta)(1+2\beta^2(1+\delta)^2)}$ which is positive if $\alpha > \frac{-1}{2\beta(1+\delta)} - \beta\delta$ (this is obviously satisfied for all positive α but also some negative ones depending on other parameter magnitudes)

When this holds, the equilibrium level of investment when there is only one district is greater than when there are two districts (but the dictator only needs one) with this particular functional form of convex costs.

My interpretation of this is that having two districts to play off each other increases the dictator's bargaining power so much that he doesn't need to affect his own outside option with investment quite as much. In the one-district bargain, the only way to affect his bargaining power is through investment; with an additional outside option of another district, the dictator can already keep much more of the regime benefit without investment. Note that the equilibrium investment with two districts is still positive (decreasing in α, increasing in β if $\alpha > \frac{\beta}{2} - \frac{1}{4\beta(1+\delta)^2}$)

Two Districts Win Two of Two

Consider the case of two districts in which the dictator must win both in order to achieve the regime benefit: if he loses either or both districts, he will not get the benefit. For simplicity, denote the probability the dictator wins one district as γ and assume the two districts are homogeneous and independent. The probability that he wins both districts without elite support is $\gamma^2 = (\frac{\alpha+\beta+\nu}{2\beta})^2$.

Second Bargaining Position

First, consider the history where the previous elite has not been coopted.

$U_D(accept\ x_2) = x_2 P(win) = x_2\gamma$

$U_D(reject) = \delta(1 - y_2)\gamma + (1 - \delta)\gamma^2$

Note the dictator only gets the regime benefit to share with the elite if he wins the election in the second district

$x_2 = \delta(1 - y_2) + (1 - \delta)\gamma$

$U_E(accept\ y_2) = y_2\gamma$

$U_E(reject) = \delta(1 - x_2)\gamma + (1 - \delta)0$

Note the elite will only have access to a portion of the regime benefit if the dictator wins the election in the second district

$y_2 = \delta(1 - x_2)$

Plug and solve:

$x_2^* = \frac{\delta+\gamma}{1+\delta}$

$y_2^* = \frac{\delta(1+\gamma)}{1+\delta}$

Note that the parties' expected utilities are still dependent on winning the other district:

$U_E(x_2^*) = (1 - \frac{\delta+\gamma}{1+\delta})\gamma$

$U_D(accept) = (\frac{\delta+\gamma}{1+\delta})\gamma$

Second, consider the history where the elite in the first bargaining position has already been coopted, leaving the dictator with remainder x_1 with which to bargain.

$U_D(accept\ x_2) = x_2 x_1$

$U_D(reject) = \delta(1 - y_2)x_1 + (1 - \delta)x_1\gamma$

$x_2 = \delta(1 - y_2) + (1 - \delta)\gamma$

$U_E(accept\ y_2) = y_2 x_1$

$U_E(reject) = \delta(1 - x_2)x_1 + (1 - \delta)0$

$y_2 = \delta(1 - x_2)$

Plug and solve:

$x_2^* = \frac{\delta+\gamma}{1+\delta}$

$y_2^* = \frac{\delta(1+\gamma)}{1+\delta}$

Note that the parties' expected utilities are still dependent on the bargain that the dictator made with the first elite:

$U_E(x_2^*) = (1 - \frac{\delta+\gamma}{1+\delta})x_1$

$U_D(accept) = (\frac{\delta+\gamma}{1+\delta})x_1$

First Bargaining Position The dictator anticipates making the aforementioned bargain with the elite in the second position.

$U_D(accept\ x_1) = x_2 x_1$

$U_D(reject) = \delta(1 - y_1)x_2 + (1 - \delta)x_2\gamma$

$x_1 = \delta(1 - y_1) + (1 - \delta)\gamma$

$U_E(accept\ y_1) = y_1$

$U_E(reject) = \delta(1 - x_1) + (1 - \delta)0$

$y_1 = \delta(1 - x_1)$

Plug and solve:

$x_1^* = \frac{\delta+\gamma}{1+\delta}$

$y_1^* = \frac{\delta(1+\gamma)}{1+\delta}$

$U_E(x_1^*) = (1 - \frac{\delta+\gamma}{1+\delta})$

$U_D(accept) = (\frac{\delta+\gamma}{1+\delta})x_2 = (\frac{\delta+\gamma}{1+\delta})^2$

Outside option form:

If the dictator and the elite in the first bargaining position fail to reach an agreement, the dictator's bargain with the elite in the second position is scaled by Ω_1, the probability he wins one district alone. This is because even if they reach an agreement, the dictator will only get the regime benefit to split with the local elite if he wins the other district, which occurs with probability Ω_1. The maximum mutual surplus that the dictator and second position elite are bargaining over is $1 * \Omega_1$. If the dictator and second elite also fail to reach an agreement, the dictator's true electoral outside option is $(\Omega_1)^2$, the probability he wins both districts alone. Thus the bargain that the dictator and second elite will come to is $(\Omega_1)^2 + \dot{\lambda}(\Omega_1 - (\Omega_1)^2) = \Omega_1(\Omega_1 + \dot{\lambda}(1 - \Omega_1))$. As we know the dictator will make a deal with the first elite, this is off path.

If the dictator and first elite do reach an agreement, the dictator's bargain with the second elite is scaled by x_1, the amount of the regime benefit the dictator is left with after the agreement with the first elite. The maximum mutual surplus over which the two parties bargain is x_1. If the dictator and second elite fail to reach an agreement in this case, the outside option is what the dictator gets if he and the second elite fail to reach an agreement: $\Omega_1 x_1$. Thus the bargain they come to is $\Omega_1 x_1 + \lambda(x_1 - \Omega_1 x_1) = x_1(\Omega_1 + \lambda(1-\Omega_1))$. Therefore in equilibrium we know that the dictator will keep $(\Omega_1 + \lambda(1-\Omega_1))$ portion of whatever is left after his bargain with the first elite. The first elite will leave him with $(\Omega_1 + \lambda(1-\Omega_1))$ between the first and second round of bargaining, so the dictator's utility at the end of both bargains after coopting both districts is $(\Omega_1 + \lambda(1-\Omega_1))^2$.

Investment

Objective function: $\max_\nu (\frac{\delta + (\frac{\alpha+\beta+\nu}{2\beta})}{1+\delta})^2 - c(\nu)$

FOC: $\frac{\alpha+\beta+2\beta\delta+\nu}{2\beta^2(1+\delta)^2} - c'(\nu) = 0$

Assume $c(\nu) = \frac{\nu^2}{2}$ such that $c'(\nu) = \nu$.

FOC: $\frac{\alpha+\beta+2\beta\delta+\nu}{2\beta^2(1+\delta)^2} = \nu$

$\nu^* = \frac{\alpha+\beta+2\beta\delta}{2\beta^2(1+\delta)^2-1}$ whether or not this is positive depends on the magnitude of β. If β is sufficiently small, the dictator should invest 0 as investment cannot be negative.

$$\nu^* = \begin{cases} 0 & \beta \leq \sqrt{\frac{1}{2}} \\ \frac{\alpha+\beta+2\beta\delta}{2\beta^2(1+\delta)^2-1} & \beta > \sqrt{\frac{1}{2}} \end{cases} \tag{1}$$

As β gets very low, the dictator's utility gets infinitely large (in the limit as $\beta \to 0$, the dictator's utility approaches ∞). Thus any investment here will just incur costs, so the dictator does not want to invest.

$\frac{\partial \nu^*}{\partial \alpha} = \frac{1}{-1+2\beta^2(1+\delta)^2}$ increasing in α

$\frac{\partial \nu^*}{\partial \beta} = -\frac{1+2\delta+2\beta(1+\delta)^2(2\alpha+\beta+2\beta\delta)}{(1-2\beta^2(1+\delta)^2)^2}$ negative: always decreasing in dictator's popularity

Three Districts

For simplicity, assume the districts all have a homogeneous win probability γ. The dictator needs at least two of the three districts to receive the benefit normalized to 1. With the micro-voting model, we know that $\gamma = \frac{\alpha+\beta}{2\beta}$ or, with investment, $\gamma_I = \frac{\alpha+\beta+\nu}{2\beta}$

Without any elites, the dictator will win the benefit with probability $\gamma^3 + (1-\gamma)3\gamma^2$

Third Bargaining Position

Consider the subgame where no prior elites have been coopted.

$U_D(accept) = \gamma(2-\gamma)x_3$

$U_D(reject) = \delta(\gamma(2-\gamma))(1-y_3) + (1-\delta)(3\gamma^2 - 2\gamma^3)$

$U_E(accept) = \gamma(2-\gamma)y_3$

$U_E(reject) = \delta(\gamma(2-\gamma)(1-x_3)) + (1-\delta)0$

$x_3 = \delta(1-y_3) + \frac{(1-\delta)(3\gamma^2-2\gamma^3)}{2\gamma-\gamma^2}$

$y_3 = \delta(1-x_3)$

$x_3^* = \frac{2\gamma^2+\gamma(\delta-3)-2\delta}{(\gamma-2)(1+\delta)}$

$y_3^* = \frac{2(\gamma-1)^2\delta}{(2-\gamma)(1+\delta)}$

Utilities: $U_D(accept|win\ other\ district) = \frac{2\gamma^2+\gamma(\delta-3)-2\delta}{(\gamma-2)(1+\delta)}$

$U_D(accept|\neg win\ other\ district) = 0$

$U_D(accept) = U_D(accept|win\ other\ district)P(win) + U_D(accept|\neg win\ other\ district)(1 - P(win))$

$U_D(accept) = (\frac{2\gamma^2+\gamma(\delta-3)-2\delta}{(\gamma-2)(1+\delta)})(2\gamma-\gamma^2)+(1-(2\gamma-\gamma^2))(0) = -\frac{\gamma(2\gamma^2+\gamma(\delta-3)-2\delta)}{1+\delta} = \frac{\gamma(2\delta+\gamma(3-\delta)-2\gamma^2)}{1+\delta}$

$U_E(x_3^*|win\ other\ district) = -\frac{2(\gamma-1)^2}{(\gamma-2)(1+\delta)} = \frac{2(\gamma-1)^2}{(2-\gamma)(1+\delta)}$

$U_E(x_3^*|\neg win\ other\ district) = 0$

$U_E(x_3^*) = U_E(x_3^*|win\ other\ district)P(win) + U_E(x_3^*|\neg win\ other\ district)(1 - P(win))$

$U_E(x_3^*) = \frac{2(\gamma-1)^2}{(2-\gamma)(1+\delta)}(2\gamma - \gamma^2) + (1 - (2\gamma - \gamma^2))(0) = \frac{2\gamma(\gamma-1)^2}{1+\delta}$

Check: sum to $1(2\gamma - \gamma^2)$

One prior elite coopted: dictator has x_2 or x_1 (I will use x_2 for simplicity) to bargain with but will not have it with certainty if he doesn't coopt this elite...

$U_D(accept) = x_3x_2$

$U_D(reject) = \delta(1 - y_3)x_2 + (1 - \delta)[x_2(2\gamma - \gamma^2) + (1 - (2\gamma - \gamma^2))(0)]$

$U_E(accept) = y_3x_2$

$U_E(reject) = \delta(1 - x_3)x_2 + (1 - \delta)0$

$y_3 = \delta(1 - x_3)$

$x_3 = \delta(1 - y_3) + (1 - \delta)(2\gamma - \gamma^2)$

$x_3^* = \frac{2\gamma-\gamma^2+\delta}{1+\delta}$

$y_3^* = \frac{(\gamma-1)^2\delta}{1+\delta}$

$U_D(accept) = \frac{2\gamma-\gamma^2+\delta}{1+\delta}x_2$

$U_E(x_3^*) = \frac{(\gamma-1)^2}{1+\delta}x_2$

Check: sum to x_2

Two prior elites coopted: dictator has x_2x_1 with certainty regardless of his bargain with the third elite

$U_d(accept) = x_3x_2x_1$

$U_D(reject) = \delta(1 - y_3)x_2x_1 + (1 - \delta)x_2x_1$

$U_E(accept) = y_3x_2x_1$

$U_E(reject) = \delta(1 - x_3)x_2x_1 + 0$

$x_3^* = 1$, $y_3^* = 0$ as the third elite is superfluous

$U_D(accept) = x_2x_1$

$U_E(x_3^*) = 0$

Second Bargaining Position

One prior elite coopted:

If the dictator makes a successful bargain with elite 2, he will be left with $x_1x_2x_3$ where $x_3 = 1$

If the dictator does not make a successful bargain with elite 2, he will make a deal with the third elite and end up with x_1x_3 where $x_3 = \frac{2\gamma-\gamma^2+\delta}{1+\delta}$ so his utility will be $\frac{2\gamma-\gamma^2+\delta}{1+\delta}x_1$

$U_D(Accept) = x_1x_2(1)$

$U_D(Reject) = \delta(1-y_2)x_1 + (1-\delta)(\frac{2\gamma-\gamma^2+\delta}{1+\delta})x_1$

$U_E(Accept) = y_2x_1$

$U_E(Reject) = \delta(1-x_2)x_1 + (1-\delta)(0)$

$x_2^* = \frac{2\gamma-\gamma^2+\delta(2+\delta)}{(1+\delta)^2}$

$y_2^* = \frac{(\gamma-1)^2\delta}{(1+\delta)^2}$

$U_D(Accept) = \frac{2\gamma-\gamma^2+\delta(2+\delta)}{(1+\delta)^2}x_1$

$U_E(x_2^*) = \frac{(\gamma-1)^2}{(1+\delta)^2}x_1$

Check: sum to x_1

No prior elite coopted:

If the dictator makes a successful bargain with elite 2, he will also bargain with elite 3 and get $\frac{2\gamma-\gamma^2+\delta}{1+\delta}x_2$

If the dictator fails to make a successful bargain with elite 2, he will bargain with elite 3 and get $\frac{\gamma(2\delta+\gamma(3-\delta)-2\gamma^2)}{1+\delta}$ (the expected utility of bargaining with the third elite taking into account the uncertainty of winning another district without elite support)

$U_D(Accept) = \frac{2\gamma-\gamma^2+\delta}{1+\delta}x_2$

$U_D(reject) = \delta(1-y_2)\frac{2\gamma-\gamma^2+\delta}{1+\delta} + (1-\delta)(\frac{\gamma(2\delta+\gamma(3-\delta)-2\gamma^2)}{1+\delta})$

$U_E(Accept) = y_2$

$U_E(Reject) = \delta(1-x_2) + (1-\delta)0$

$x_2 = \delta(1-y_2) + \frac{\gamma(2\gamma^2+\gamma(\delta-3)-2\delta)(\delta-1)}{2\gamma-\gamma^2+\delta}$

$y_2 = \delta(1 - x_2)$

$x_2^* = \frac{-2\gamma^3+\gamma^2(3-2\delta)+4\gamma\delta+\delta^2}{(1+\delta)(2\gamma-\gamma^2+\delta)}$

$y_2^* = \frac{\delta(\gamma-1)^2(2\gamma+\delta)}{(1+\delta)(2\gamma-\gamma^2+\delta)}$

$U_D(Accept) = \frac{2\gamma-\gamma^2+\delta}{1+\delta} * \frac{-2\gamma^3+\gamma^2(3-2\delta)+4\gamma\delta+\delta^2}{(1+\delta)(2\gamma-\gamma^2+\delta)} = -\frac{(\gamma(2\gamma-3)-\delta)(\gamma+\delta)}{(1+\delta)^2} = \frac{(\gamma(3-2\gamma)+\delta)(\gamma+\delta)}{(1+\delta)^2}$

$U_E(x_2^*) = 1 - \frac{-2\gamma^3+\gamma^2(3-2\delta)+4\gamma\delta+\delta^2}{(1+\delta)(2\gamma-\gamma^2+\delta)} = \frac{(\gamma-1)^2(2\gamma+\delta)}{(1+\delta)(2\gamma-\gamma^2+\delta)}$

Check: $U_D + U_{E2} + U_{E3}$ sums to 1

First Bargaining Position

If the dictator makes a successful bargain with elite 1, he will then bargain with the later elites and get $x_1 x_2 x_3$ where $x_3 = 1$ and $x_2 = \frac{2\gamma-\gamma^2+\delta(2+\delta)}{(1+\delta)^2}$ so his utility will be $\frac{2\gamma-\gamma^2+\delta(2+\delta)}{(1+\delta)^2}x_1$ If the dictator does not complete a successful bargain with elite 1, he will still coopt elites 2 and 3 and get $\frac{(\gamma(3-2\gamma)+\delta)(\gamma+\delta)}{(1+\delta)^2}$

$U_D(accept) = \frac{2\gamma-\gamma^2+\delta(2+\delta)}{(1+\delta)^2}x_1$

$U_D(reject) = \delta(1 - y_1)\frac{2\gamma-\gamma^2+\delta(2+\delta)}{(1+\delta)^2} + (1-\delta)\frac{(\gamma(3-2\gamma)+\delta)(\gamma+\delta)}{(1+\delta)^2}$

$U_E(accept) = y_1$

$U_E(reject) = \delta(1 - x_1) + (1 - \delta)(0)$

$x_1 = \delta(1 - y_1) + (1 - \delta)\frac{(\gamma(3-2\gamma)+\delta)(\gamma+\delta)}{2\gamma-\gamma^2+\delta(2+\delta)}$

$x_1^* = \frac{\delta^2-\delta(\gamma-3)+\gamma(2-2\gamma)}{(1+\delta)(2+\delta-\gamma)}$

$y_1^* = \frac{\delta(\gamma-1)^2(2\gamma+2\delta+1)}{(1+\delta)^3}$

$U_D(Accept) = \frac{\delta^2-\delta(\gamma-3)+\gamma(2-2\gamma)}{(1+\delta)(2+\delta-\gamma)} * \frac{2\gamma-\gamma^2+\delta(2+\delta)}{(1+\delta)^2} = \frac{(3+\delta-2\gamma)(\delta+\gamma)^2}{(1+\delta)^3}$

$U_E(x_1^*) = \frac{2(\gamma-1)^2}{(1+\delta)(2+\delta-\gamma)}$

Check: $U_D + U_{E1} + U_{E2}$ sums to 1

Equilibrium

The dictator will complete successful bargains with elites in the first and second bargaining positions. He will reject any offer less than 1 from the third elite, the elite is indifferent between offering 1 and continuing the bargain as he will get 0 regardless.

$U_D(accept x_1^*, accept x_2^*) = \frac{(3-2\gamma+\delta)(\gamma+\delta)^2}{(1+\delta)^3}$

$\frac{\partial}{\partial\gamma} U_D(accept x_1^*, accept x_2^*) = \frac{6(1-\gamma)(\gamma+\delta)}{(1+\delta)^3}$ positive, increasing in γ

$\frac{\partial}{\partial\delta} U_D(accept x_1^*, accept x_2^*) = \frac{6(\gamma-1)^2(\gamma+\delta)}{(1+\delta)^4}$ positive, increasing in δ

Plug in micro probability $\gamma = \frac{\alpha+\beta}{2\beta}$

$U_D(accept x_1^*, accept x_2^*) = \frac{(3-2(\frac{\alpha+\beta}{2\beta})+\delta)(\frac{\alpha+\beta}{2\beta}+\delta)^2}{(1+\delta)^3} = \frac{(\alpha+\beta+2\beta\delta)^2(\beta(2+\delta)-\alpha)}{4\beta^3(1+\delta)^3}$

$\frac{\partial}{\partial\alpha} U_D(accept x_1^*, accept x_2^*) = \frac{3(\beta-\alpha)(\alpha+\beta+2\beta\delta)}{4\beta^3(1+\delta)^3}$ positive; increasing in α the dictator's ex ante popularity

$\frac{\partial}{\partial\beta} U_D(accept x_1^*, accept x_2^*) = \frac{3\alpha(\alpha-\beta)(\alpha+\beta+2\beta\delta)}{4\beta^4(1+\delta)^3}$ sign depends on the sign of α. If $\alpha > 0$, decreasing in β. If $\alpha < 0$, increasing in β

$U_{E1}(x_1^*) = \frac{2(\gamma-1)^2}{(2-\gamma+\delta)(1+\delta)}$

$\frac{\partial}{\partial\gamma} U_{E1}(x_1^*) = \frac{2(1-\gamma)(\gamma-3-2\delta)}{(1+\delta)(2-\gamma+\delta)^2}$

$\gamma - 3 - 2\delta > 0$ this will never hold, thus the entire derivative is negative. decreasing in γ

$\frac{\partial}{\partial\delta} U_{E1}(x_1^*) = \frac{2(\gamma-1)^2(\gamma-3-2\delta)}{(1+\delta)^2(2-\gamma+\delta)^2}$ negative, decreasing in δ

Plug in micro probability $\gamma = \frac{\alpha+\beta}{2\beta}$

$U_{E1}(x_1^*) = \frac{2(\gamma-1)^2}{(2-\gamma+\delta)(1+\delta)} = \frac{2((\frac{\alpha+\beta}{2\beta})-1)^2}{(2-\frac{\alpha+\beta}{2\beta}+\delta)(1+\delta)} = \frac{(\alpha-\beta)^2}{\beta(1+\delta)(\beta(3+2\delta)-\alpha)}$

$\frac{\partial}{\partial\alpha} U_{E1}(x_1^*) = \frac{(\beta-\alpha)(\beta(5+4\delta)-\alpha)}{\beta(1+\delta)(\alpha-\beta(3+2\delta))^2}$ I'm getting that this is positive (increasing in α) which doesn't really make sense...

$\frac{\partial}{\partial\beta} U_{E1}(x_1^*) = \frac{\alpha(\alpha-\beta)(\alpha-\beta(5+4\delta))}{\beta^2(1+\delta)(\alpha-\beta(3+2\delta))^2}$ sign depends on α. If α is positive, increasing in β. If α is negative, decreasing in β.

$U_{E2}(x_2^*) = \frac{(\gamma-1)^2}{(1+\delta)^2} x_1 = \frac{(\gamma-1)^2}{(1+\delta)^2} * \frac{-2\gamma^2+\gamma(3-\delta)+\delta(3+\delta)}{(1+\delta)(2-\gamma+\delta)} = \frac{(\gamma-1)^2(3-2\gamma+\delta)(\gamma+\delta)}{(2-\gamma+\delta)(1+\delta)^3}$

$\frac{\partial}{\partial\gamma} U_{E2}(x_2^*) = \frac{2(\gamma-1)(-3+3\gamma^3+4\delta+5\delta^2+\delta^3-3\gamma^2(4+\delta)+\gamma(13+2\delta-2\delta^2))}{(1+\delta)^3(2-\gamma+\delta)^2}$

this is not easy to sign, but from pictures the utility function looks nonmonotonic in γ if δ is sufficiently low. When δ gets close to .5 or higher, E2's utility is decreasing in γ

$\frac{\partial}{\partial\delta} U_{E2}(x_2^*) = -\frac{(\gamma-1)^2(-3+3\gamma^3+4\delta+5\delta^2+\delta^3-3\gamma^2(4+\delta)+\gamma(13+2\delta-2\delta^2))}{(1+\delta)^4(2-\gamma+\delta)^2}$

this is not easy to sign, but from pictures the utility function looks nonmonotonic in δ when γ is sufficiently low, then decreasing in δ when γ is around .3 or greater.

Plug in micro probability $\gamma = \frac{\alpha+\beta}{2\beta}$

$U_{E2}(x_2^*) = \frac{((\frac{\alpha+\beta}{2\beta})-1)^2(3-2\frac{\alpha+\beta}{2\beta}+\delta)(\frac{\alpha+\beta}{2\beta}+\delta)}{(2-\frac{\alpha+\beta}{2\beta}+\delta)(1+\delta)^3} = \frac{(\alpha-\beta)^2(\alpha+\beta+2\beta\delta)(\beta(2+\delta)-\alpha)}{4\beta^3(1+\delta)^3(\beta(3+2\delta)-\alpha)}$

$\frac{\partial}{\partial\alpha}$ this is pretty gnarly, but pretty sure it is negative. decreasing in α

$\frac{\partial}{\partial\beta}$ similarly difficult, but looks like it depends on α as above. If $\alpha > 0$ increasing in β; if $\alpha < 0$ decreasing in β

If I simplify and just look at $(1 - x_2^*)$, the elite's portion of x_1, it is easier to work with. $\frac{\partial}{\partial\alpha}(1 - x_2^*) = \frac{\alpha-\beta}{2\beta^2(1+\delta)^2}$ which is negative, decreasing in α. But x_1^* the dictator's share of the first bargain is increasing in α

$$U_{E3}(x_3^*) = 0$$

$$U_{E1} - U_{E2} = \frac{(\gamma-1)^2(2+2\gamma^2+\gamma(\delta-3)+\delta+\delta^2)}{(2-\gamma+\delta)(1+\delta)^3}$$

from plots, this quantity is always positive and always decreasing in δ, always decreasing in γ

$U_{E1} - U_{E2} = \frac{(\alpha-\beta)^2(\alpha^2+\alpha\beta(\delta-1)+\beta^2(2+3\delta+2\delta^2))}{4\beta^3(1+\delta)^3(\beta(3+2\delta)-\alpha)}$

Always positive; this is decreasing in α; change in β depends on α: if α is positive, increasing in β; if α negative, decreasing in β

Three District Investment

Using the micro-founded model with uniform valence, the dictator's ex ante probability of winning a district without elite support is $\frac{\beta+\alpha}{2\beta}$ where $\beta > 0$ and $\alpha > 0$ indicates the dictator is idiosyncratically popular ($\alpha < 0$ indicates the dictator is idiosyncratically unpopular)

If the dictator invests in popular policies (redistribution), his valence distribution is increased by a factor of $\nu \in [0, \infty)$ in all districts $\sim U[-\beta+\alpha+\nu, \beta+\alpha+\nu]$. His probability of winning any one district is $\frac{\beta+\alpha+\nu}{2\beta}$

Investment is costly. The cost of investment $c(\nu)$ is a function of the amount of investment ν, such that $c'(\nu) > 0$, $c'' > 0$, and $c(0) = 0$.

The dictator's utility from the three-district (must win 2) rubinstein alternating-offers bargain: $\frac{(3-2\gamma+\delta)(\gamma+\delta)^2}{(1+\delta)^3}$

Substitute in the micro-founded win probability: $\frac{(3-2(\frac{\beta+\alpha+\nu}{2\beta})+\delta)((\frac{\beta+\alpha+\nu}{2\beta})+\delta)^2}{(1+\delta)^3} - c(\nu)$

Confirm concavity: $\frac{\partial^2}{\partial\nu^2}\frac{(3-2(\frac{\beta+\alpha+\nu}{2\beta})+\delta)((\frac{\beta+\alpha+\nu}{2\beta})+\delta)^2}{(1+\delta)^3} - c(\nu) = -\frac{3(\alpha+\beta\delta+\nu)}{2\beta^3(1+\delta)^3} - c''(\nu)$

Recall c is convex by definition, so $-c$ is concave

The first term is negative iff $\alpha + \beta\delta + \nu > 0$. By definition, β, δ, ν are all positive. If the dictator is idiosyncratically unpopular, α may be negative. It must hold that $\beta\delta+\nu > -\alpha$ for the dictator's objective function to be strictly concave. If α is extremely large and negative relative to the other parameters, the dictator is so unpopular that any investment in his popular support will not make a difference in his likelihood of winning. In the limit where $\delta \to 1$, this requirement that $\beta + \nu \geq -\alpha$ ensures that the dictator's probability of winning any individual district is greater than or equal to 0 (and thus a proper probability).

Assume the dictator is sufficiently popular.

$$\max_\nu \frac{(3-2(\frac{\beta+\alpha+\nu}{2\beta})+\delta)((\frac{\beta+\alpha+\nu}{2\beta})+\delta)^2}{(1+\delta)^3} - c(\nu)$$

$$\frac{\partial}{\partial\nu}\frac{(3-2(\frac{\beta+\alpha+\nu}{2\beta})+\delta)((\frac{\beta+\alpha+\nu}{2\beta})+\delta)^2}{(1+\delta)^3} - c(\nu) = \frac{3(\beta-\alpha-\nu)(\alpha+\beta+2\beta\delta+\nu)}{4\beta^3(1+\delta)^3} - c'(\nu) = 0 \text{ (FOC)}$$

Let's assume a functional form for the convex cost. Let $c(\nu) = \frac{\nu^2}{2}$ so $c'(\nu) = \nu$ and $c''(\nu) = 1$.

FOC: $\frac{3(\beta-\alpha-\nu)(\alpha+\beta+2\beta\delta+\nu)}{4\beta^3(1+\delta)^3} - \nu = 0$

$\nu^* = -\frac{1}{3}\beta^3(1+\delta)^3(2 + \frac{3\alpha}{(\beta^3(1+\delta)^3)} + \frac{3\delta}{(\beta^2(1+\delta)^3)} - \sqrt{4 + \frac{(9+12\beta(1+\delta)(\alpha+\beta\delta))}{(\beta^4(1+\delta)^4)}})$

$\frac{\partial\nu^*}{\partial\alpha} = \frac{2}{\sqrt{4+\frac{(9+12\beta(1+\delta)(\alpha+\beta\delta))}{(\beta^4(1+\delta)^4)}}} - 1$ this could be positive or negative depending on parameter magnitudes. Specifically positive if $\alpha < \frac{-3-4\beta^2\delta-4\beta^2\delta^2}{4\beta(1+\delta)}$; if α is sufficiently low, investment is increasing in the dictator's popularity in that district. Else investment is decreasing in the dictator's popularity.

$\frac{\partial \nu^*}{\partial \beta} = -\delta - 2\beta^2(1+\delta)^3 - \frac{(2(3+\beta(1+\delta)(3\alpha+2\beta\delta)))}{(\beta^2(1+\delta)\sqrt{4+\frac{(9+12\beta(1+\delta)(\alpha+\beta\delta))}{(\beta^4(1+\delta)^4)}})} + \beta^2(1+\delta)^3\sqrt{4+\frac{(9+12\beta(1+\delta)(\alpha+\beta\delta))}{(\beta^4(1+\delta)^4)}}$ can be positive or negative depending on magnitudes. Positive for extreme values (high and low) of α, negative in the middle. Specifically investment is decreasing in β if $\alpha \in \frac{1}{6}(-4\beta\delta + \frac{-1-2\delta}{\beta(1+\delta)^3} - \frac{2}{\beta+\beta\delta} - \sqrt{\frac{(3+6\delta+4\delta^2)(\delta+2\beta^2(1+\delta)^3)^2}{\beta^2(1+\delta)^6}}), \frac{1}{6}(-4\beta\delta + \frac{-1-2\delta}{\beta(1+\delta)^3} - \frac{2}{\beta+\beta\delta} + \sqrt{\frac{(3+6\delta+4\delta^2)(\delta+2\beta^2(1+\delta)^3)^2}{\beta^2(1+\delta)^6}}))$

Surplus

Social Surplus $= 1 - P(win\neg elite) = 1-(3\gamma^2-2\gamma^3) = 1-(3(\frac{\alpha+\beta}{2\beta})^2 - 2(\frac{\alpha+\beta}{2\beta})^3) = \frac{(\alpha-\beta)^2(\alpha+2\beta)}{4\beta^3}$

$\frac{\partial}{\partial\alpha} = \frac{3(\alpha-\beta)(\alpha+\beta)}{4\beta^3}$ the surplus is decreasing in α

$\frac{\partial}{\partial\beta} = \frac{3(\alpha\beta^2-\alpha^3)}{4\beta^4}$ if α is positive, increasing in β. If α is negative, non-monotonic in β

The above social surplus is the difference between a coalition with no elites and a coalition with sufficient elites (2 or more) to win with certainty. Theoretically (off path) the dictator could coopt fewer elites (1) and still generate a surplus as having one district with certainty reduces the overall uncertainty the dictator faces. In this case, the surplus is the difference between a 1-elite coalition and a 0-elite coalition: $2\gamma - \gamma^2 - (3\gamma^2 - 2\gamma^3) = 2\gamma(\gamma-1)^2 = 2\frac{\alpha+\beta}{2\beta}(\frac{\alpha+\beta}{2\beta}-1)^2 = \frac{(\alpha-\beta)^2(\alpha+\beta)}{4\beta^3}$

Obviously the surplus of going from no elites to sufficient elites to win with certainty is greater than the surplus of going from none to one: while having one elite in the coalition makes the dictator more likely to succeed, achieving the regime benefit is still uncertain.

I used another definition of surplus above that I want to try again here: just from the dictator's perspective, the surplus is the difference in his expected utility from using the elite intermediary option relative to no elites (off path).

$U_D(elites) = \frac{(3-2\gamma+\delta)(\gamma+\delta)^2}{(1+\delta)^3}$

$U_D(\neg elites) = 3\gamma^2 - 2\gamma^3$

Dictator surplus $= \frac{(3-2\gamma+\delta)(\gamma+\delta)^2}{(1+\delta)^3} - (3\gamma^2 - 2\gamma^3) = \frac{\delta(\gamma-1)^2(6\gamma+\delta^2(1+2\gamma)+\delta(3+6\gamma))}{(1+\delta)^3}$

positive (obviously) and less than the social surplus as the dictator's expected utility from using the elite intermediaries is less as it excludes the portion that the elites take

This quantity is decreasing in α, change in β depends on sign of α. If α is negative, decreasing in β; if α is positive, increasing in β

Two of Three Districts Discussion

Consider a state with three homogeneous districts of which the dictator needs the support of at least two in order to remain in power and receive the regime benefit, normalized to 1. If he does not coopt any of the elites, the dictator wins each district via mass support with probability from the micro voting model, $\gamma = \frac{\alpha+\beta}{2\beta}$. Thus his overall electoral outside option $\Omega_3 = \gamma^3 + 3\gamma^2(1-\gamma)$. This electoral outside option is increasing in the dictator's popularity and decreasing in electoral uncertainty when the dictator is popular, increasing in the electoral uncertainty when the dictator is unpopular. The dictator's outside option when he must win two of three districts to achieve the regime benefit is greater than his likelihood of winning two of two when he goes it alone in the election. However, his electoral outside option for two of three is worse than one of two ($\Omega_3 < \Omega_2'$). While, like one of two, there is a superfluous elite, needing to win two districts to keep the regime does depress his outside option. Coopting two elites will make the dictator win with certainty; adding the third elite does not increase his benefit. While there may be other reasons a dictator wants an oversized coalition or super-majority support (see Groseclose and Snyder 1996, Magaloni 2006), these specific incentives are outside the scope of the current model.

Lemma 15. *In equilibrium, the elite in the third bargaining position will never receive a portion of the regime benefit greater than 0.*

Lemma 16. *Let the probability the dictator wins each district equal* γ. *When the dictator must win two of three districts, for any investment* $\nu \geq 0$, *the dictator coopts all three elites; the first elite takes* $\frac{2}{(1+\delta)(2+\delta-\gamma)(1+2\gamma)}(1-\Omega_3)$, *the second elite takes* $\frac{(3+\delta-2\gamma)(\delta+\gamma)}{(1+\delta)^3(2+\delta-\gamma)(1+2\gamma)}(1-\Omega_3)$, *the third elite takes 0. The dictator is left with* $\Omega_3 + \lambda_3(1-\Omega_3)$. *If he were to make counter offers to each elite, they would be* $(\frac{\delta(\gamma-1)^2(2\gamma+2\delta+1)}{(1+\delta)^3}, (\frac{(\gamma-1)^2\delta}{(1+\delta)^2})x_1, 0)$

The portion of the surplus that utilizing the elites instead of using the election alone that the dictator keeps, λ_3, is $(\frac{\delta^2(3+\delta)}{(1+\delta)^3} + \frac{6\delta\gamma}{(1+\delta)^3(1+2\gamma)})$. As one elite is superfluous and the dictator can use this to play the elites off of one another to strengthen his bargaining position, the portion of the surplus that he keeps in this institutional arrangement is greater than his portion when he needed two of two districts ($\lambda_3 > \lambda_2$). However, this portion is less than what he keeps in the one of two institutional configuration ($\lambda'_2 > \lambda_3$).

Which bargaining position is best for an elite? Obviously being in the third position leaves the elite worst off as she will maintain a utility of 0. The first bargaining position is always best for the elite, but the difference between the utilities of being in the first or second positions depends on the other parameters. The first position's advantage over the second decreases as the dictator is more popular. When the dictator is *ex ante* popular, the first position advantage is increasing in the uncertainty of the mass election; when the dictator is unpopular, the first position advantage is decreasing in the uncertainty of the mass election.

Proposition 10. *When the dictator must win two of three districts and anticipates the equilibrium elite offers of* $(\frac{\delta^2+\delta(3-\gamma)+2\gamma(1-\gamma)}{(1+\delta)(2+\delta-\gamma)}, \frac{(\delta+\gamma)(\delta^2+\delta(3-\gamma)+2\gamma(1-\gamma))}{(1+\delta)^3}, 1)$ *he makes optimal investment which satisfies the first order condition* $\frac{3(\beta-\alpha-\nu)(\alpha+\beta+2\beta\delta+\nu)}{4\beta^3(1+\delta)^3} - c'(\nu) = 0$.

Corollary 5. *When the dictator must win two of three districts, anticipates the equilibrium elite offers of* $(\frac{\delta^2+\delta(3-\gamma)+2\gamma(1-\gamma)}{(1+\delta)(2+\delta-\gamma)}, \frac{(\delta+\gamma)(\delta^2+\delta(3-\gamma)+2\gamma(1-\gamma))}{(1+\delta)^3}, 1)$, *he makes optimal investment* $\nu^* = -\frac{1}{3}\beta^3(1+\delta)^3(2 + \frac{3\alpha}{(\beta^3(1+\delta)^3)} + \frac{3\delta}{(\beta^2(1+\delta)^3)} - \sqrt{4 + \frac{(9+12\beta(1+\delta)(\alpha+\beta\delta))}{(\beta^4(1+\delta)^4)}})$

Much like investment in the one of two district configuration, investment is decreasing in the dictator's district popularity α unless α is sufficiently low. For extreme values of α

(high and low), investment in increasing in β; for middling β, investment is decreasing in β. These comparative statics are similar to the one of two institutional configuration in the dictator's incentive to invest more when he is unpopular and the sensitivity of investment to electoral uncertainty depends on the dictator's popularity.

Takeaways Homogeneous Districts

One of One

Outside option: $\frac{\alpha+\beta}{2\beta}$

$\frac{\partial}{\partial\alpha} = \frac{1}{2\beta}$

$\frac{\partial}{\partial\beta} = -\frac{\alpha}{2\beta^2}$ sign depends on α: positive if α is negative. In general, increasing β makes the slope flatter, but also extends the domain of α. So when α is negative, the upward shift of the utility curve when β increases overwhelms the flattening slope

Dictator's utility through bargaining (no investment): $\frac{\alpha+\beta+2\beta\delta}{2\beta(1+\delta)}$ this is the same as no bargaining if $\delta = 0$. Slope (still linear) is lower (flatter) as δ increases

Optimal investment without bargaining: $\frac{1}{2\beta}$

Optimal investment with bargaining: $\frac{1}{2\beta(1+\delta)}$

both decreasing in β

The difference between the dictator's utility (with optimal investment) when bargaining with the elites relative to no elites is the greatest when β is low.

Two of Two

Outside Option: $(\frac{\alpha+\beta}{2\beta})^2$

$\frac{\partial}{\partial\alpha} = \frac{\alpha+\beta}{2\beta^2}$ this is always positive

$\frac{\partial}{\partial\beta} = -\frac{\alpha(\alpha+\beta)}{2\beta^3}$ positive if α is negative, negative if α is positive. Same as above: the slope of the dictator's utility flattens as β increases, but when α is negative the changing domain is the overwhelming effect

Dictator's utility through bargaining (no investment): $\frac{(\delta+\frac{\alpha+\beta}{2\beta})^2}{(1+\delta)^2}$

Optimal investment without bargaining: $\frac{\alpha+\beta}{2\beta^2-1}$ investment is always increasing in α, but change in β depends on parameter magnitudes. Increasing in β if $\alpha < -\frac{1+2\beta^2}{4\beta}$, else decreasing in β

Optimal investment with bargaining: $\frac{\alpha+\beta+2\beta\delta}{2\beta^2(1+\delta)^2-1}$ similarly always increasing in α (for investment greater than 0). Increasing in β if α is sufficiently low, specifically if $\alpha < -\frac{(1+2\delta)(1+2\beta^2(1+\delta)^2))}{4\beta(1+\delta)^2}$ else decreasing in β

This follows similar logic as above: the slope of investment (in α) is flattening as β increases, but for very low α the change in domain overwhelms this effect and increasing β has a positive effect on investment

One of Two

Outside Option: $2(\frac{\alpha+\beta}{2\beta}) - (\frac{\alpha+\beta}{2\beta})^2 = \frac{(3\beta-\alpha)(\alpha+\beta)}{4\beta^2}$

As β increases, the outside option curve (in α) gets flatter. For high α, the curve is already very flat so the changing domain in increasing β dominates

Dictator's utility through bargaining (no investment): $\frac{(\alpha+\beta+2\beta\delta)(\beta(3+2\delta)-\alpha)}{4\beta^2(1+\delta)^2}$

Optimal Investment (no bargaining): $\frac{\beta-\alpha}{1+2\beta^2}$

decreasing in α, decreasing in β unless α is sufficiently high. In particular, if $\alpha > \frac{2\beta^2-1}{4\beta}$, increasing in β

Optimal Investment (with bargaining): $\frac{\beta-\alpha}{1+2\beta^2(1+\delta)^2}$

decreasing in α, β partial depends on relative parameters. Positive if α is sufficiently large, specifically if $\alpha > \frac{\beta^2(1+\delta)^2-1}{2\beta(1+\delta)^2}$.

Two of Three

Outside Option: $\frac{(2\beta-\alpha)(\alpha+\beta)^2}{4\beta^3}$

Dictator's utility through bargaining (no investment): $\frac{(\alpha+\beta+2\beta\delta)^2(\beta(2+\delta)-\alpha)}{4\beta^3(1+\delta)^3}$

Optimal investment (no bargaining): $\frac{1}{3}(-3\alpha - 2\beta^3 + \beta\sqrt{9 + 12\alpha\beta + 4\beta^4})$

Optimal Investment (with bargaining): $-\frac{1}{3}\beta^3(1+\delta)^3(2+\frac{3\alpha}{(\beta^3(1+\delta)^3)}+\frac{3\delta}{(\beta^2(1+\delta)^3)}-\sqrt{4+\frac{(9+12\beta(1+\delta)(\alpha+\beta\delta))}{(\beta^4(1+\delta)^4)}})$

depends on parameters; increasing in α if α is sufficiently low, else decreasing in α. Increasing in β for low and high α, decreasing in β for middling α (but if δ is closer to 1, just increasing for high α else decreasing)

Two Heterogeneous Districts

There are three possible ways to define district heterogeneity in terms of the dictator's electoral chances[25]:

- same dispersion, different mean st $V_1 \sim U[-\beta+\alpha_1, \beta+\alpha_1]$ and $V_2 \sim U[-\beta+\alpha_2, \beta+\alpha_2]$
- same mean, difference dispersion st $V_1 \sim U[-\beta_1+\alpha, \beta_1+\alpha]$ and $V_2 \sim U[-\beta_2+\alpha, \beta_2+\alpha]$
- different mean and dispersion st $V_1 \sim U[-\beta_1+\alpha_1, \beta_1+\alpha_1]$ and $V_2 \sim U[-\beta_2+\alpha_2, \beta_2+\alpha_2]$

I will utilize generic probabilities γ_1 and γ_2 as placeholders.

One of Two

Full Result:

If $\chi < \hat{\chi}_{1,2} = 1 - \frac{1}{4\beta_1\beta_2(1+\delta)^2}$, the dictator invests

$$\hat{\nu}_1^* = \frac{\beta_1 + 4\beta_1\beta_2(1+\delta)^2(\alpha_2 - \beta_2 + \chi\beta_1) + \alpha_1(-1 - 4\beta_1\beta_2\chi(1+\delta)^2)}{1 + 8\beta_1\beta_2(1+\delta)^2(\chi + 2\beta_1\beta_2(1+\delta)^2(\chi^2 - 1))}$$

$$\hat{\nu}_2^* = \frac{\beta_2 + 4\beta_1\beta_2(1+\delta)^2(\alpha_1 - \beta_1 + \chi\beta_2) + \alpha_2(-1 - 4\beta_1\beta_2(1+\delta)^2\chi)}{1 + 8\beta_1\beta_2(1+\delta)^2(\chi + 2\beta_1\beta_2(1+\delta)^2(\chi^2 - 1))}$$

in district 1 and district 2, respectively. If $\chi \geq \hat{\chi}_{1,2}$, the dictator will invest either $\hat{\nu}_1 = \frac{\beta_2 - \alpha_2}{4\beta_1\beta_2(1+\delta)^2}$ or $\hat{\nu}_2 = \frac{\beta_1 - \alpha_1}{4\beta_1\beta_2(1+\delta)^2}$: if and only if $\alpha_1 - \alpha_2 > \beta_1 - \beta_2$ invest $\hat{\nu}_1$ in district 1, else invest $\hat{\nu}_2$ in district 2.

[25] there also could be heterogeneity in investment costs or something like that, but I am going to focus on differences in win probability

Without elite support, the dictator's probability of winning at least one of the two districts and achieving the regime benefit is $\gamma_1 + \gamma_2 - \gamma_1\gamma_2$.

When the dictator must win one of two districts, the bargain follows as described above, but now his outside electoral option when bargaining with the second elite and not coopting the first is $\gamma_1 + \gamma_2 - \gamma_1\gamma_2$.

The dictator's expected utility of this bargaining game is $U_D(x_1^*, x_2^*) = \frac{2\delta+\delta^2+\gamma_1+\gamma_2-\gamma_1\gamma_2}{(1+\delta)^2}$ Note that even if the second district in the bargaining order is is better for the dictator in terms of outside win probability ($\gamma_2 > \gamma_1$), the dictator will prefer to come to an agreement with the first elite as the superfluous bargaining partner will always make him better off. Further note that even if the dictator could choose the bargaining order (instead of nature), he will be indifferent between either bargaining order as his expected utility is the same and he has no preference over which local elite shares more of the regime benefit.

For investment, we need to specify the heterogeneous win probabilities. I will use the most heterogeneous where $V_1 \sim U[-\beta_1 + \alpha_1, \beta_1 + \alpha_1]$ and $V_2 \sim U[-\beta_2 + \alpha_2, \beta_2 + \alpha_2]$. The dictator's objective function is

$\max_{\nu_1,\nu_2} \frac{2\delta+\delta^2+\frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}+\frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}-\frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}*\frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}}{(1+\delta)^2} - c(\nu_1,\nu_2)$

$\frac{\partial}{\partial\nu_1} = \frac{-\alpha_2+\beta_2-\nu_2}{4\beta_1\beta_2(1+\delta)^2} - c'(\nu_1,\nu_2)$

$\frac{\partial}{\partial\nu_2} = \frac{-\alpha_1+\beta_1-\nu_1}{4\beta_1\beta_2(1+\delta)^2} - c'(\nu_1,\nu_2)$

Assume $c(\nu_1,\nu_2) = \frac{\nu_1^2}{2} + \frac{\nu_2^2}{2} + \chi\nu_1\nu_2$ where $\chi \geq 0$ indicates the complementarity of costs from investing in each district. $\frac{\partial}{\partial\nu_1}c(\nu_1,\nu_2) = \nu_1 + \chi\nu_2$ and $\frac{\partial}{\partial\nu_2}c(\nu_1,\nu_2) = \nu_2 + \chi\nu_1$

First Order Conditions:

$\frac{\partial}{\partial\nu_1} = \frac{-\alpha_2+\beta_2-\nu_2}{4\beta_1\beta_2(1+\delta)^2} - (\nu_1 + \chi\nu_2) = 0$

$\frac{\partial}{\partial\nu_2} = \frac{-\alpha_1+\beta_1-\nu_1}{4\beta_1\beta_2(1+\delta)^2} - (\nu_2 + \chi\nu_1) = 0$

Second Partials:

$\frac{\partial^2}{\partial\nu_1^2} = -1$

$\frac{\partial^2}{\partial \nu_2^2} = -1$

$\frac{\partial^2}{\partial \nu_1 \nu_2} = -\chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2}$

Hessian: $\begin{bmatrix} -1 & -\chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2} \\ -\chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2} & -1 \end{bmatrix}$

$$\begin{bmatrix} -1 & -\chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2} \\ -\chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2} & -1 \end{bmatrix} - \lambda I = \begin{bmatrix} -1-\lambda & -\chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2} \\ -\chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2} & -1-\lambda \end{bmatrix}$$

Characteristic polynomial: $(-1-\lambda)^2 - (-\chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2})^2 = 0$

$\lambda = -1 \pm (\chi + \frac{1}{4\beta_1\beta_2(1+\delta)^2})$

If eigenvalues are all non-negative, positive semi-definite. If eigenvalues are all non-positive, negative semi-definite.

$-1 - \chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2} > 0$ is not possible by definition of χ, so the hessian is not positive semi-definite.

$-1 - \chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2} < 0$ always holds by definition of χ

Check $-1 + \chi + \frac{1}{4\beta_1\beta_2(1+\delta)^2} < 0$

$\chi < 1 - \frac{1}{4\beta_1\beta_2(1+\delta)^2}$ if this holds, the hessian is negative semi-definite

To be negative definite, it must be the case that the determinant of the negative semi-definite matrix is not zero.

$(-1)^2 - (-\chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2})^2 \neq 0$

This would be violated if $\chi = 1 - \frac{1}{4\beta_1\beta_2(1+\delta)^2}$ which is already ruled out by the eigenvalue condition on χ, therefore the hessian is negative definite and the solution $(\nu_1^* = \frac{-\alpha_2+\beta_2-\nu_2}{4\beta_1\beta_2(1+\delta)^2} - \chi\nu_2, \nu_2^* = \frac{-\alpha_1+\beta_1-\nu_1}{4\beta_1\beta_2(1+\delta)^2} - \chi\nu_1)$ is a local maximum if $\chi < 1 - \frac{1}{4\beta_1\beta_2(1+\delta)^2}$

CLOSED FORM

$\nu_1^* = \frac{-\alpha_2+\beta_2-\nu_2}{4\beta_1\beta_2(1+\delta)^2} - \chi\nu_2, \nu_2^* = \frac{-\alpha_1+\beta_1-\nu_1}{4\beta_1\beta_2(1+\delta)^2} - \chi\nu_1$

For all the following comparative statics, assume that $\beta_1\beta_2 > \frac{1}{4(1+\delta)^2}$, the districts are sufficiently uncertain. Further, $\chi \in [0, 1-\frac{1}{4\beta_1\beta_2(1+\delta)^2})$ so investing in both districts is optimal.

$\nu_1^* = \frac{\beta_1+4\beta_1\beta_2(1+\delta)^2(\alpha_2-\beta_2+\chi\beta_1)+\alpha_1(-1-4\beta_1\beta_2\chi(1+\delta)^2)}{1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$

Comparative Statics:

$\frac{\partial \nu_1^*}{\partial \alpha_1} = \frac{-1-4\beta_1\beta_2(1+\delta)^2\chi}{1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$ numerator and denominator are negative, so derivative is positive. Investment in district 1 is increasing in the dictator's ex ante popularity in district 1.

$\frac{\partial \nu_1^*}{\partial \alpha_2} = \frac{4\beta_1\beta_2(1+\delta)^2}{1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$ negative. Investment in district 1 is decreasing in the dictator's ex ante popularity in district 2.

$\frac{\partial \nu_1^*}{\partial \beta_1} =$

$\frac{(1+4\beta_2(1+\delta)^2(\alpha_2+(\alpha_1+2\beta_1)\chi-16\alpha_2\beta_2^2\beta_1^2(1+\delta)^4(-1+\chi^2)+16\beta_2^3\beta_1^2(1+\delta)^4(-1+\chi^2)+16\alpha_1\beta_2^2\beta_1^2(1+\delta)^4\chi(-1+\chi^2)+\beta_2(-1+4\beta_1(1+\delta)^2(-2\alpha_1+\beta_1+(}{(1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(-1+\chi^2)))^2}$

Limits summary:

$\beta_2 > \beta_1$

$\alpha_1 >> \alpha_2$ derivative negative

$\alpha_1 > \alpha_2$ negative

$\alpha_1 < \alpha_2$ negative

$\alpha_1 << \alpha_2$ positive

$\beta_1 > \beta_2$

$\alpha_1 >> \alpha_2$ derivative negative

$\alpha_1 > \alpha_2$ negative

$\alpha_1 < \alpha_2$ negative

$\alpha_1 << \alpha_2$ positive

the derivative is either decreasing in χ or non-monotonic in χ depending on parameter magnitudes

As χ approaches its maximum (in the limit), $\frac{\partial \nu_1^*}{\partial \beta_1}$ is negative if $\alpha_1 - \alpha_2 > \beta_1 - \beta_2$

$$\frac{\partial \nu_1^*}{\partial \beta_2} = \frac{(4\beta_1(1+\delta)^2(\alpha_2+(\alpha_1-\beta_1)\chi-16\alpha_2\beta_1^2\beta_2^2(1+\delta)^4(-1+\chi^2)+8\beta_1\beta_2^2(1+\delta)^2\chi(-1+2(\alpha_1-\beta_1)\beta_1(1+\delta)^2(-1+\chi^2))+2\beta_2(-1+4(\alpha_1-\beta_1)\beta_1(1+\delta)^2(-1+\chi^2)))}{(1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(-1+\chi^2)))^2}$$

Limits summary:

$\beta_2 > \beta_1$

$\alpha_1 >> \alpha_2$ negative

$\alpha_1 > \alpha_2$ positive

$\alpha_1 < \alpha_2$ positive

$\alpha_1 << \alpha_2$ positive

$\beta_1 > \beta_2$

$\alpha_1 >> \alpha_2$ negative

$\alpha_1 > \alpha_2$ negative

$\alpha_1 < \alpha_2$ positive

$\alpha_1 << \alpha_2$ positive

Generally increasing in χ but non-monotonic in χ for certain magnitudes.

As χ approaches its max, derivative is negative if $\alpha_1 - \alpha_2 > \beta_1 - \beta_2$

For ν_2^*:

$\nu_1^* = \frac{-\alpha_2+\beta_2-\nu_2}{4\beta_1\beta_2(1+\delta)^2} - \chi\nu_2, \nu_2^* = \frac{-\alpha_1+\beta_1-\nu_1}{4\beta_1\beta_2(1+\delta)^2} - \chi\nu_1$

$\nu_2^* = \frac{\beta_2+4\beta_1\beta_2(1+\delta)^2(\alpha_1-\beta_1+\chi\beta_2)+\alpha_2(-1-4\beta_1\beta_2(1+\delta)^2\chi)}{1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$

$\frac{\partial \nu_2^*}{\partial \alpha_2} = \frac{-1-4\beta_1\beta_2(1+\delta)^2\chi}{(1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(-1+\chi^2))}$ numerator and denominator are negative so derivative is positive. Investment in district 2 is increasing in the dictator's ex ante popularity in district 2.

$\frac{\partial \nu_2^*}{\partial \alpha_1} = \frac{4\beta_1\beta_2(1+\delta)^2}{1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$ negative. Investment in district 2 is decreasing in the dictator's ex ante popularity in district 1.

$\frac{\partial \nu_2^*}{\partial \beta_2} =$

$$\frac{(1+4\beta_1(1+\delta)^2(\alpha_1+(\alpha_2+2\beta_2)\chi-16\alpha_1\beta_1^2\beta_2^2(1+\delta)^4(-1+\chi^2)+16\beta_1^3\beta_2^2(1+\delta)^4(-1+\chi^2)+16\alpha_2\beta_1^2\beta_2^2(1+\delta)^4\chi(-1+\chi^2)+\beta_1(-1+4\beta_2(1+\delta)^2(-2\alpha_2+\beta_2+(}{(1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(-1+\chi^2)))^2}$$

Limits summary:

$\beta_2 > \beta_1$

$\alpha_1 >> \alpha_2$ positive

$\alpha_1 > \alpha_2$ positive

$\alpha_1 < \alpha_2$ negative

$\alpha_1 << \alpha_2$ negative

$\beta_1 > \beta_2$

$\alpha_1 >> \alpha_2$ positive

$\alpha_1 > \alpha_2$ negative

$\alpha_1 < \alpha_2$ negative

$\alpha_1 << \alpha_2$ negative

$\frac{\partial \nu_2^*}{\partial \beta_1} =$

$$\frac{(4\beta_2(1+\delta)^2(\alpha_1+(\alpha_2-\beta_2)\chi-16\alpha_1\beta_1^2\beta_2^2(1+\delta)^4(-1+\chi^2)+8\beta_1^2\beta_2(1+\delta)^2\chi(-1+2(\alpha_2-\beta_2)\beta_2(1+\delta)^2(-1+\chi^2))+2\beta_1(-1+4(\alpha_2-\beta_2)\beta_2(1+\delta)^2(-1+\chi^2)))}{(1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(-1+\chi^2)))^2}$$

Limits summary:

$\beta_2 > \beta_1$

$\alpha_1 >> \alpha_2$ positive

$\alpha_1 > \alpha_2$ positive

$\alpha_1 < \alpha_2$ positive

$\alpha_1 << \alpha_2$ negative

$\beta_1 > \beta_2$

$\alpha_1 >> \alpha_2$ positive

$\alpha_1 > \alpha_2$ positive

$\alpha_1 < \alpha_2$ negative

$\alpha_1 << \alpha_2$ negative

In general, investment is increasing in the dictator's popularity in the district (invest more where you are ahead). How district uncertainty affects investment depends on the relative parameters: increasing uncertainty in the district decreases investment in that district unless the dictator is severely unpopular in the district (relative to the other district). When the dictator is very behind, district uncertainty actually benefits him, so his investment in increasing in the uncertainty of the district only in that case.

if χ is too big....

Options (possible corner solutions): $(\nu_1, 0), (0, \nu_2), (0, 0)$

$\frac{2\delta+\delta^2+\frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}+\frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}-\frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}*\frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}}{(1+\delta)^2} - \frac{\nu_1^2}{2} - \frac{\nu_2^2}{2} - \chi\nu_1\nu_2$

$(\nu_1, 0) : \frac{2\delta+\delta^2+\frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}+\frac{\alpha_2+\beta_2}{2\beta_2}-\frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}*\frac{\alpha_2+\beta_2}{2\beta_2}}{(1+\delta)^2} - \frac{\nu_1^2}{2}$

maximizing wrt ν_1 yields FOC $\frac{-\alpha_2+\beta_2-4\beta_1\beta_2(1+\delta)^2\nu_1}{4\beta_1\beta_2(1+\delta)^2} = 0$

$\hat{\nu_1} = \frac{\beta_2-\alpha_2}{4\beta_1\beta_2(1+\delta)^2}$

$U_D(\hat{\nu_1}, 0) = \frac{2\delta+\delta^2+\frac{\alpha_1+\beta_1+(\frac{\beta_2-\alpha_2}{4\beta_1\beta_2(1+\delta)^2})}{2\beta_1}+\frac{\alpha_2+\beta_2}{2\beta_2}-\frac{\alpha_1+\beta_1+(\frac{\beta_2-\alpha_2}{4\beta_1\beta_2(1+\delta)^2})}{2\beta_1}*\frac{\alpha_2+\beta_2}{2\beta_2}}{(1+\delta)^2} - \frac{(\frac{\beta_2-\alpha_2}{4\beta_1\beta_2(1+\delta)^2})^2}{2} =$
$1 + \frac{(\alpha_2-\beta_2)(\alpha_2-\beta_2(1+8(\alpha_1-\beta_1)\beta_1(1+\delta)^2))}{32\beta_1^2\beta_2^2(1+\delta)^4}$

$(0, \nu_2) : \frac{2\delta+\delta^2+\frac{\alpha_1+\beta_1}{2\beta_1}+\frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}-\frac{\alpha_1+\beta_1}{2\beta_1}*\frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}}{(1+\delta)^2} - \frac{\nu_2^2}{2}$

maximizing wrt ν_2 yields FOC $\frac{-\alpha_1+\beta_1-4\beta_1\beta_2(1+\delta)^2\nu_2}{4\beta_1\beta_2(1+\delta)^2}$

$\hat{\nu_2} = \frac{\beta_1-\alpha_1}{4\beta_1\beta_2(1+\delta)^2}$

$U_D(0,\hat{\nu_2}) = \frac{2\delta+\delta^2+\frac{\alpha_1+\beta_1}{2\beta_1}+\frac{\alpha_2+\beta_2+(\frac{\beta_1-\alpha_1}{4\beta_1\beta_2(1+\delta)^2})}{2\beta_2}-\frac{\alpha_1+\beta_1}{2\beta_1}*\frac{\alpha_2+\beta_2+(\frac{\beta_1-\alpha_1}{4\beta_1\beta_2(1+\delta)^2})}{2\beta_2}}{(1+\delta)^2} - \frac{(\frac{\beta_1-\alpha_1}{4\beta_1\beta_2(1+\delta)^2})^2}{2}$
$= 1 + \frac{(\alpha_1-\beta_1)(\alpha_1-\beta_1(1+8(\alpha_2-\beta_2)\beta_2(1+\delta)^2))}{32\beta_1^2\beta_2^2(1+\delta)^4}$

$(0,0) : \frac{2\delta+\delta^2+\frac{\alpha_1+\beta_1}{2\beta_1}+\frac{\alpha_2+\beta_2}{2\beta_2}-\frac{\alpha_1+\beta_1}{2\beta_1}*\frac{\alpha_2+\beta_2}{2\beta_2}}{(1+\delta)^2}$
$U_D(0,0) = 1 + \frac{(\alpha_1-\beta_1)(-\alpha_2+\beta_2)}{4\beta_1\beta_2(1+\delta)^2}$

$(0,0)$ is dominated by investment in a single district, so just compare only investing in district one versus only investing in district two:

Compare $U_D(\hat{\nu_1},0), U_D(0,\hat{\nu_2})$:

$U_D(\hat{\nu_1},0) - U_D(0,\hat{\nu_2}) = \frac{-\alpha_1^2+\alpha_2^2+2\alpha_1\beta_1-\beta_1^2+\beta_2^2-2\alpha_2\beta_2}{32\beta_1^2\beta_2^2(1+\delta)^4}$

This quantity is positive (and thus investing in district 1 is preferred) iff $(\beta_1-\alpha_1+\beta_2-\alpha_2)(\alpha_1-\beta_1+\beta_2-\alpha_2)$. Because $\beta > \alpha$ by assumption for a proper probability, district 1 is preferred iff $\beta_2-\alpha_2 > \beta_1-\alpha_1$

$\alpha_1-\alpha_2 > \beta_1-\beta_2$

Note that if the uncertainty in each district is the same or district 2 is more uncertain, being more popular in district 1 guarantees the dictator will invest there. Even when he is not ahead in district 1, the dictator may invest there depending on the relative uncertainties (i.e. if district 2 is very uncertain)

General takeaway:

If the complementarity of costs is sufficiently low, the dictator will invest in both districts. If the complementarity of costs is not sufficiently low, the dictator will invest in one district (no investment is dominated). He will invest in the less uncertain district in which his investment will make a greater impact (higher mean ex ante popularity, lower dispersion).

Comparative statics for individual district investment (when χ is too high)

$\hat{\nu_1} = \frac{\beta_2-\alpha_2}{4\beta_1\beta_2(1+\delta)^2}$

$\frac{\partial\hat{\nu_1}}{\partial\alpha_2} = \frac{-1}{4\beta_1\beta_2(1+\delta)^2} < 0$ investment is decreasing in the dictator's popularity in the other district

$\frac{\partial \hat{\nu_1}}{\partial \beta_2} = \frac{\alpha_2}{4\beta_1\beta_2^2(1+\delta)^2}$ sign depends on whether α_2 is positive or negative. If α_2 is positive, investment in district 1 is increasing in the uncertainty of district 2. If α_2 is negative (in which case uncertainty in district 2 benefits the dictator), investment in district 1 is decreasing in the uncertainty of district 2.

$\frac{\partial \hat{\nu_1}}{\partial \beta_1} = \frac{\alpha_2-\beta_2}{4\beta_1^2\beta_2(1+\delta)^2} < 0$ investment decreasing in the dispersion of the district

$$\hat{\nu_2} = \frac{\beta_1-\alpha_1}{4\beta_1\beta_2(1+\delta)^2}$$

$\frac{\partial \hat{\nu_2}}{\partial \alpha_1} = \frac{-1}{4\beta_1\beta_2(1+\delta)^2} < 0$ investment is decreasing in the dictator's popularity in the other district

$\frac{\partial \hat{\nu_2}}{\partial \beta_2} = \frac{\alpha_1-\beta_1}{4\beta_1\beta_2^2(1+\delta)^2} < 0$ investment decreasing in the dispersion of the district

$\frac{\partial \hat{\nu_2}}{\partial \beta_1} = \frac{\alpha_1}{4\beta_1^2\beta_2(1+\delta)^2}$ sign depends on whether α_1 is positive or negative

One of Two with no Elites

Comparison of investment with and without bargaining protocol

One of two with no bargaining: Without elite support, the dictator's probability of winning at least one of the two districts and achieving the regime benefit is $\gamma_1 + \gamma_2 - \gamma_1\gamma_2$.

$\frac{\alpha_1+\beta_1+\nu_1}{2\beta_1} + \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2} - \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1} * \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}$

What is optimal investment without bargaining?

$\max_{\nu_1,\nu_2} \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1} + \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2} - \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1} * \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2} - c(\nu_1,\nu_2)$

$\max_{\nu_1,\nu_2} \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1} + \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2} - \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1} * \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2} - \frac{\nu_1^2}{2} - \frac{\nu_2^2}{2} - \chi\nu_1\nu_2$

$$\frac{\partial}{\partial \nu_1} = \frac{1}{2\beta_1} - \frac{\alpha_2+\beta_2\nu_2}{4\beta_1\beta_2} - \chi\nu_2 - \nu_1$$

$\nu_1^* = \frac{1}{2\beta_1} - \frac{\alpha_2+\beta_2\nu_2}{4\beta_1\beta_2} - \chi\nu_2$

$\frac{\partial}{\partial \nu_2} = \frac{1}{2\beta_2} - \frac{\alpha_1+\beta_1\nu_1}{4\beta_1\beta_2} - \chi\nu_1 - \nu_2$

$\nu_2^* = \frac{1}{2\beta_2} - \frac{\alpha_1+\beta_1\nu_1}{4\beta_1\beta_2} - \chi\nu_1$

Confirm concavity/interior conditions

$\frac{\partial}{\partial \nu_1^2} = -1$

$\frac{\partial}{\partial \nu_2^2} = -1$

$\frac{\partial}{\partial \nu_1 \nu_2} = -\frac{1}{4\beta_1\beta_2} - \chi$

Hessian: $\begin{bmatrix} -1 & -\frac{1}{4\beta_1\beta_2} - \chi \\ -\frac{1}{4\beta_1\beta_2} - \chi & -1 \end{bmatrix}$

Determinant: $(-1)^2 - (-\frac{1}{4\beta_1\beta_2} - \chi)^2$ which is positive if $\chi < 1 - \frac{1}{4\beta_1\beta_2}$

Assume $\chi \in [0, 1 - \frac{1}{4\beta_1\beta_2})$ If this holds, interior solution (invest in both districts) is optimal.

Closed Form

$\nu_1^* = \frac{\beta_1 + 4\beta_1\beta_2(\alpha_2 - \beta_2 + \beta_1\chi) - \alpha_1(1 + 4\beta_1\beta_2\chi)}{(1 + 4\beta_1\beta_2(\chi - 1))(1 + 4\beta_1\beta_2(1 + \chi))}$

$\nu_2^* = \frac{\beta_2 + 4\beta_1\beta_2(\alpha_1 - \beta_1 + \beta_2\chi) - \alpha_2(1 + 4\beta_1\beta_2\chi)}{(1 + 4\beta_1\beta_2(\chi - 1))(1 + 4\beta_1\beta_2(\chi + 1))}$

Basic Comparative Statics

ν_1^* is increasing in α_1, the dictator's popularity in district 1

ν_1^* is decreasing in α_2, the dictator's popularity in the other district

ν_2^* is increasing in α_2, the dictator's popularity in district 2

ν_2^* is decreasing in α_1, the dictator's popularity in the other district

Corner Solutions: $(\nu_1, 0), (0, \nu_2), (0, 0)$

$(\nu_1, 0)$

$\max_{\nu_1} \frac{\alpha_1 + \beta_1 + \nu_1}{2\beta_1} + \frac{\alpha_2 + \beta_2}{2\beta_2} - \frac{\alpha_1 + \beta_1 + \nu_1}{2\beta_1} * \frac{\alpha_2 + \beta_2}{2\beta_2} - \frac{\nu_1^2}{2} - \chi\nu_1 * 0$

$\nu_1^* = \frac{\beta_2 - \alpha_2}{4\beta_1\beta_2}$

$U_D(\nu_1^*, 0) = \frac{\alpha_2^2 - 2\alpha_2(1 + 4(\alpha_1 - \beta_1)\beta_1)\beta_2 + (1 + 8\beta_1(\alpha_1 + 3\beta_1))\beta_2^2}{32\beta_1^2\beta_2^2}$

$(0, \nu_2)$

$\max_{\nu_2} \frac{\alpha_1+\beta_1}{2\beta_1} + \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2} - \frac{\alpha_1+\beta_1}{2\beta_1} * \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2} - \frac{\nu_2^2}{2} - \chi * 0 * \nu_2$

$\nu_2^* = \frac{\beta_1-\alpha_1}{4\beta_1\beta_2}$

$U_D(0, \nu_2^*) = \frac{\alpha_1^2-2\alpha_1\beta_1(1+4\alpha_2\beta_2-4\beta_2^2)+\beta_1^2(1+8\alpha_2\beta_2+24\beta_2^2)}{32\beta_2^2\beta_1^2}$

$(0, 0)$

$U_D = \frac{\alpha_1+\beta_1}{2\beta_1} + \frac{\alpha_2+\beta_2}{2\beta_2} - \frac{\alpha_1+\beta_1}{2\beta_1} * \frac{\alpha_2+\beta_2}{2\beta_2}$

$= \frac{\alpha_1(\beta_2-\alpha_2)+\beta_1(\alpha_2+3\beta_2)}{4\beta_1\beta_2}$

$(0, 0)$ is dominated

Whether $(\nu_1^*, 0)$ or $(0, \nu_2^*)$ is preferred depends on relative parameter magnitudes.

$U_D(\nu_1^*, 0) - U_D(0, \nu_2^*) = \frac{(\alpha_1-\alpha_2+\beta_2-\beta_1)(\beta_1-\alpha_1+\beta_2-\alpha_2)}{32\beta_1^2\beta_2^2}$

$\beta > \alpha$ within district by proper probability. So investing in district 1 is preferred if $(\alpha_1 - \alpha_2 + \beta_2 - \beta_1) > 0$

Invest in district 1 if $\frac{\alpha_1-\alpha_2}{\beta_1-\beta_2} > 1$ which is the same condition as with the elite intermediary

The point of this is to compare investment/utility with and without the bargaining protocol.

Assume χ is sufficiently low so the dictator invests in both districts

Investment in district 1 with bargaining: $\nu_1^* = \frac{\beta_1+4\beta_1\beta_2(1+\delta)^2(\alpha_2-\beta_2+\chi\beta_1)+\alpha_1(-1-4\beta_1\beta_2\chi(1+\delta)^2)}{1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$

Investment in district 1 without bargaining: $\nu_1^* = \frac{\beta_1+4\beta_1\beta_2(\alpha_2-\beta_2+\beta_1\chi)-\alpha_1(1+4\beta_1\beta_2\chi)}{(1+4\beta_1\beta_2(\chi-1))(1+4\beta_1\beta_2(1+\chi))}$

For $\delta > 0$, investment is higher without bargaining

Note that the χ (cost complementarity) at which the dictator no longer wants to invest in both districts differs with and without elite intermediaries. The range of χs for which the dictator invests in both districts is larger when there are elite intermediaries to bargain with.

Investing in the superfluous district (since he only needs one of two) is more attractive in the elite bargaining case because he is not just increasing his probability of winning but increasing his bargaining position.

Two of Two

Full Result:

If $\chi < \hat{\chi}_{2,2} = 1 + \frac{1}{4\beta_1\beta_2(1+\delta)^2}$, the dictator invests

$$\hat{\nu}_1^* = -\frac{\alpha_1 + \beta_1 + 2\beta_1\delta + 4\beta_1\beta_2(1+\delta)^2(\alpha_2 + \beta_2 + 2\beta_2\delta) - 4\beta_1\beta_2(1+\delta)^2(\alpha_1 + \beta_1 + 2\beta_1\delta)\chi}{1 + 8\beta_1\beta_2(1+\delta)^2(-\chi + 2\beta_1\beta_2(1+\delta)^2(-1+\chi^2))}$$

$$\hat{\nu}_2^* = -\frac{\alpha_2 + \beta_2 + 2\beta_2\delta + 4\beta_1\beta_2(1+\delta)^2(\alpha_1 + \beta_1 + 2\beta_1\delta) - 4\beta_1\beta_2(1+\delta)^2(\alpha_2 + \beta_2 + 2\beta_2\delta)\chi}{1 + 8\beta_1\beta_2(1+\delta)^2(-\chi + 2\beta_1\beta_2(1+\delta)^2(-1+\chi^2))}$$

in district 1 and district 2, respectively. If $\chi \geq \hat{\chi}_{2,2}$, the dictator will invest either $\hat{\nu_1} = \frac{\alpha_2+\beta_2+2\beta_2\delta}{4\beta_1\beta_2(1+\delta)^2}$ or $\hat{\nu_2} = \frac{\alpha_1+\beta_1+2\beta_1\delta}{4\beta_1\beta_2(1+\delta)^2}$: if and only if $\alpha_2 - \alpha_1 > (\beta_1 - \beta_2)(1+2\delta)$, invest in $\hat{\nu_1}$ in district 1, else invest $\hat{\nu_2}$ in district 2.

Without elite support, the dictator's probability of winning both districts and achieving the regime benefit is $\gamma_1\gamma_2$. The bargain follows as described above, but now the dictator's true electoral outside option is $\gamma_1\gamma_2$ and if he fails to coopt the first elite, his expected utility is scaled by the probability that he wins the first district through unsupported electoral means as he needs both to get the benefit.

The dictator's objective function is

$\max_{\nu_1,\nu_2}(\frac{\delta+\gamma_2}{1+\delta})(\frac{\delta+\gamma_1}{1+\delta}) - c(\nu_1,\nu_2)$

Substituting the heterogeneous district win probabilities:

$\max_{\nu_1,\nu_2}(\frac{\delta+\frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}}{1+\delta})(\frac{\delta+\frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}}{1+\delta}) - c(\nu_1,\nu_2)$

Assume $c(\nu_1,\nu_2) = \frac{\nu_1^2}{2} + \frac{\nu_2^2}{2} + \chi\nu_1\nu_2$ where $\chi \geq 0$ indicates the complementarity of costs from investing in each district. $\frac{\partial}{\partial\nu_1}c(\nu_1,\nu_2) = \nu_1 + \chi\nu_2$ and $\frac{\partial}{\partial\nu_2}c(\nu_1,\nu_2) = \nu_2 + \chi\nu_1$

First Order Conditions:

$\frac{\partial}{\partial \nu_1} = \frac{\alpha_2+\beta_2+2\beta_2\delta+\nu_2}{4\beta_1\beta_2(1+\delta)^2} - \nu_1 - \chi\nu_2 = 0$

$\frac{\partial}{\partial \nu_2} = \frac{\alpha_1+\beta_1+2\beta_1\delta+\nu_1}{4\beta_1\beta_2(1+\delta)^2} - \nu_2 - \chi\nu_1 = 0$

Second Partials:

$\frac{\partial^2}{\partial \nu_1^2} = -1$

$\frac{\partial^2}{\partial \nu_2^2} = -1$

$\frac{\partial^2}{\partial \nu_1 \nu_2} = \frac{1}{4\beta_1\beta_2(1+\delta)^2} - \chi$

$$\text{Hessian: } \begin{bmatrix} -1 & \frac{1}{4\beta_1\beta_2(1+\delta)^2} - \chi \\ \frac{1}{4\beta_1\beta_2(1+\delta)^2} - \chi & -1 \end{bmatrix}$$

Determinant: $(-1)^2 - (\frac{1}{4\beta_1\beta_2(1+\delta)^2} - \chi)^2$ which is positive if $\chi < 1 + \frac{1}{4\beta_1\beta_2(1+\delta)^2}$

As $-1 < 0$ and the determinant of the Hessian is positive if χ is sufficiently low, $\nu_1^* = \frac{\alpha_2+\beta_2+2\beta_2\delta+\nu_2}{4\beta_1\beta_2(1+\delta)^2} - \chi\nu_2, \nu_2^* = \frac{\alpha_1+\beta_1+2\beta_1\delta+\nu_1}{4\beta_1\beta_2(1+\delta)^2} - \chi\nu_1)$ is a local maximum

Note this cutoff in χ is higher than the cutoff when he needed to win one of two. There is a larger range of cost complementarities for which investing in both districts is optimal.

Closed form:

$\nu_1^* = \frac{\alpha_2+\beta_2+2\beta_2\delta+\nu_2}{4\beta_1\beta_2(1+\delta)^2} - \chi\nu_2, \nu_2^* = \frac{\alpha_1+\beta_1+2\beta_1\delta+\nu_1}{4\beta_1\beta_2(1+\delta)^2} - \chi\nu_1)$

$\nu_1^* = -\frac{\alpha_1+\beta_1+2\beta_1\delta+4\beta_1\beta_2(1+\delta)^2(\alpha_2+\beta_2+2\beta_2\delta)-4\beta_1\beta_2(1+\delta)^2(\alpha_1+\beta_1+2\beta_1\delta)\chi}{1+8\beta_1\beta_2(1+\delta)^2(-\chi+2\beta_1\beta_2(1+\delta)^2(-1+\chi^2))}$

$\frac{\partial \nu_1^*}{\partial \alpha_1} = -\frac{1-4\beta_1\beta_2(1+\delta)^2\chi}{1+8\beta_1\beta_2(1+\delta)^2(-\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$ mostly negative (maybe positive for super low βs and low χ

$\frac{\partial \nu_1^*}{\partial \alpha_2} = -\frac{4\beta_1\beta_2(1+\delta)^2}{1+8\beta_1\beta_2(1+\delta)^2(-\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$ mostly positive (maybe negative for super low βs but they probably violate my conditions

$\frac{\partial \nu_1^*}{\partial \beta_1} = \frac{((-1-2\delta-4\alpha_2\beta_2(1+\delta)^2-4\beta_2^2(1+\delta)^2(1+2\delta)+4\beta_2\chi(1+\delta)^2(\alpha_1+2\beta_1+4\beta_1\delta))(1+8\beta_1\beta_2(-\chi(1+\delta)^2+2\beta_1\beta_2(-1+\chi^2)(1+\delta)^4))-8\beta_2(-\chi(1+\delta)^2-}{(1+8\beta_1\beta_2(-\chi(1+\delta)^2+2\beta_1\beta_2(-1+\chi^2)(1+\delta)}$

whether this is positive or negative depends on relative parameter sizes (like one of two)

$\frac{\partial \nu_1^*}{\partial \beta_2} = \frac{(4\beta_1(1+\delta)^2(-\chi(\alpha_1+\beta_1+2\beta_1\delta)+\alpha_2(-1+16\beta_1^2\beta_2^2(-1+\chi^2)(1+\delta)^4)-8\beta_1\beta_2^2\chi(1+\delta)^2(-1-2\delta+2\beta_1(-1+\chi^2)(1+\delta)^2(\alpha_1+\beta_1+2\beta_1\delta))+2\beta_2(-1-}{(1+8\beta_1\beta_2(-\chi(1+\delta)^2+2\beta_1\beta_2(-1+\chi^2)(1+\delta)^4))^2}$

this is positive or negative depends on relative parameter sizes (like one of two) but mostly positive

Closed form

$\nu_2^* = \frac{\alpha_2(-1+4\beta_1\beta_2\chi(1+\delta)^2)+\beta_2(-1-2\delta-4\beta_1(1+\delta)^2(\alpha_1+(\beta_1-\beta_2\chi)(1+2\delta)))}{1+8\beta_1\beta_2(1+\delta)^2(-\chi+2\beta_1\beta_2(1+\delta)^2(-1+\chi^2))}$

$\frac{\partial \nu_2^*}{\partial \alpha_2} = \frac{-1+4\beta_1\beta_2(1+\delta)^2\chi}{1+8\beta_1\beta_2(1+\delta)^2(-\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$ negative

$\frac{\partial \nu_2^*}{\partial \alpha_1} = -\frac{4\beta_1\beta_2(1+\delta)^2}{1+8\beta_1\beta_2(1+\delta)^2(-\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$ positive

$\frac{\partial \nu_2^*}{\partial \beta_2} = \frac{(-1-2\delta+4\beta_1(1+\delta)^2(-16\alpha_2\beta_1^2\beta_2^2\chi(-1+\chi^2)(1+\delta)^4+16\beta_1^3\beta_2^2(-1+\chi^2)(1+\delta)^4(1+2\delta)+\chi(-\alpha_2+2\beta_2+4\beta_2\delta)+\alpha_1(-1+16\beta_1^2\beta_2^2(-1+\chi^2)(1+\delta)^4)}{1+8\beta_1\beta_2(1+\delta)^2(-\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$

whether this is positive or negative depends on relative parameter sizes (like one of two)

$\frac{\partial \nu_2^*}{\partial \beta_1} = \frac{(4\beta_2(1+\delta)^2(-\chi(\alpha_2+\beta_2+2\beta_2\delta)+\alpha_1(-1+16\beta_1^2\beta_2^2(-1+\chi^2)(1+\delta])^4)-8\beta_1^2\beta_2\chi(1+\delta)^2(-1-2\delta+2\beta_2(-1+\chi^2)(1+\delta)^2(\alpha_2+\beta_2+2\beta_2\delta))+2\beta_1(-1-}{1+8\beta_1\beta_2(1+\delta)^2(-\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$

whether this is positive or negative depends on relative parameter sizes (like one of two)

What if χ is too big?

Possible corner solutions: $(\nu_1, 0), (0, \nu_2), (0, 0)$

$(\nu_1, 0) : (\frac{\delta+\frac{\alpha_2+\beta_2}{2\beta_2}}{1+\delta})(\frac{\delta+\frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}}{1+\delta}) - \frac{\nu_1^2}{2}$

maximize wrt ν_1 yields FOC: $\frac{\alpha_2+\beta_2+2\beta_2\delta}{4\beta_1\beta_2(1+\delta)^2} - \nu_1 = 0$

$\hat{\nu_1} = \frac{\alpha_2+\beta_2+2\beta_2\delta}{4\beta_1\beta_2(1+\delta)^2}$

$U_D(\hat{\nu_1}, 0) = \frac{(\alpha_2+\beta_2+2\beta_2\delta)(\alpha_2+\beta_2+2\beta_2\delta+8\beta_1\beta_2(1+\delta)^2(\alpha_1+\beta_1+2\beta_1\delta))}{32\beta_1^2\beta_2^2(1+\delta)^4}$

$(0, \nu_2) : (\frac{\delta+\frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}}{1+\delta})(\frac{\delta+\frac{\alpha_1+\beta_1}{2\beta_1}}{1+\delta}) - \frac{\nu_2^2}{2}$

maximize wrt ν_2 yields FOC: $\frac{\alpha_1+\beta_1+2\beta_1\delta}{4\beta_1\beta_2(1+\delta)^2} - \nu_2 = 0$

$\hat{\nu_2} = \frac{\alpha_1+\beta_1+2\beta_1\delta}{4\beta_1\beta_2(1+\delta)^2}$

$U_D(0, \hat{\nu_2}) = \frac{(\alpha_1+\beta_1+2\beta_1\delta)(\alpha_1+\beta_1+2\beta_1\delta+8\beta_1\beta_2(1+\delta)^2(\alpha_2+\beta_2+2\beta_2\delta))}{32\beta_1^2\beta_2^2(1+\delta)^4}$

$$(0,0):(\frac{\delta+\frac{\alpha_2+\beta_2}{2\beta_2}}{1+\delta})(\frac{\delta+\frac{\alpha_1+\beta_1}{2\beta_1}}{1+\delta})$$

$$U_D(0,0)=\frac{(\alpha_1+\beta_1+2\beta_1\delta)(\alpha_2+\beta_2+2\beta_2\delta)}{4\beta_1\beta_2(1+\delta)^2}$$

$(0,0)$ is dominated by investment in a single district, so just compare investment in district one versus investment in district two:

$U_D(\hat{\nu_1},0)-U_D(0,\hat{\nu_2})=$

$$\frac{(\alpha_2+\beta_2+2\beta_2\delta)(\alpha_2+\beta_2+2\beta_2\delta+8\beta_1\beta_2(1+\delta)^2(\alpha_1+\beta_1+2\beta_1\delta))}{32\beta_1^2\beta_2^2(1+\delta)^4}-\frac{(\alpha_1+\beta_1+2\beta_1\delta)(\alpha_1+\beta_1+2\beta_1\delta+8\beta_1\beta_2(1+\delta)^2(\alpha_2+\beta_2+2\beta_2\delta))}{32\beta_1^2\beta_2^2(1+\delta)^4}$$

positive if $-(\alpha_1+\alpha_2+\beta_1+\beta_2+2\delta(\beta_1+\beta_2))(\alpha_1-\alpha_2+(\beta_1-\beta_2)(1+2\delta))>0$

The first term is always positive

Invest in district 1 if and only if

$\alpha_2-\alpha_1>(\beta_1-\beta_2)(1+2\delta)$

Comparative statics for individual district investment (χ too high)

$\hat{\nu_1}=\frac{\alpha_2+\beta_2+2\beta_2\delta}{4\beta_1\beta_2(1+\delta)^2}$

$\frac{\partial}{\partial\alpha_2}=\frac{1}{4\beta_1\beta_2(1+\delta)^2}$ positive: increasing in popularity in OTHER district

$\frac{\partial}{\partial\beta_2}=\frac{-\alpha_2}{4\beta_1\beta_2^2(1+\delta)^2}$ sign depends on sign of α_2

$\frac{\partial}{\partial\beta_1}=-\frac{\alpha_2+\beta_2+2\beta_2\delta}{4\beta_1\beta_2^2(1+\delta)^2}$ negative: decreasing in uncertainty dispersion in this district

$\hat{\nu_2}=\frac{\alpha_1+\beta_1+2\beta_1\delta}{4\beta_1\beta_2(1+\delta)^2}$

$\frac{\partial}{\partial\alpha_1}=\frac{1}{4\beta_1\beta_2(1+\delta)^2}$ positive: increasing in popularity in OTHER district

$\frac{\partial}{\partial\beta_2}=-\frac{\alpha_1+\beta_1+2\beta_1\delta}{4\beta_1^2\beta_2(1+\delta)^2}$ negative: decreasing in uncertainty dispersion in this district

$\frac{\partial}{\partial\beta_1}=\frac{-\alpha_1}{4\beta_1^2\beta_2(1+\delta)^2}$ sign depends on sign of α_1

Takeaways

When the dictator needs to win one of two districts, investments in his electoral outside options across the two districts are substitutes. The optimal investment level for one district is decreasing in the investment level for the other. When the dictator must win both districts in order to receive the regime benefit, investments in his electoral outside options across the districts are complements. The optimal investment level for one district is increasing in the investment level for the other.

Three Heterogeneous Districts

Let γ_1, γ_2, and γ_3 stand in for the dictator's probability of winning each district without elite support.

Third Bargaining Position

No Prior Districts Coopted

If no elites are coopted, the dictator's expected probability of winning at least two of the three districts is $\gamma_1\gamma_2 + \gamma_1\gamma_3 + \gamma_2\gamma_3 - 2\gamma_1\gamma_2\gamma_3$

If he coopts the third district, his probability of winning at least one of the other two districts to receive the regime benefit is $\gamma_1 + \gamma_2 - \gamma_1\gamma_2$

$U_D(accept x_3) = x_3(\gamma_1 + \gamma_2 - \gamma_1\gamma_2)$

$U_D(reject) = \delta(1 - y_3)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2) + (1 - \delta)(\gamma_1\gamma_2 + \gamma_1\gamma_3 + \gamma_2\gamma_3 - 2\gamma_1\gamma_2\gamma_3)$

$U_E(accept) = y_3((\gamma_1 + \gamma_2 - \gamma_1\gamma_2)$ as she will only get a portion of the regime benefit if the dictator succeeds

$U_E(reject) = \delta(1 - x_3)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2) + (1 - \delta)(0)$

$x_3^* = \frac{(\gamma_1\gamma_2+\gamma_1\gamma_3+\gamma_2\gamma_3-2\gamma_1\gamma_2\gamma_3)+\delta(\gamma_1+\gamma_2-\gamma_1\gamma_2)}{(1+\delta)(\gamma_1+\gamma_2-\gamma_1\gamma_2)}$

$y_3^* = \delta(1 - \frac{(\gamma_1\gamma_2+\gamma_1\gamma_3+\gamma_2\gamma_3-2\gamma_1\gamma_2\gamma_3)+\delta(\gamma_1+\gamma_2-\gamma_1\gamma_2)}{(1+\delta)(\gamma_1+\gamma_2-\gamma_1\gamma_2)})$

One Previous Elite Coopted: District 1

$U_D(accept) = x_1 x_3$

$U_D(reject) = \delta(1-y_3)x_1 + (1-\delta)x_1(\gamma_2 + \gamma_3 - \gamma_2\gamma_3)$ the probability the dictator wins district 2, 3, or both without elite support

$U_E(accept) = y_3 x_1$

$U_E(reject) = \delta(1-x_3)x_1 + (1-\delta)(0)$

$x_3^* = \frac{\delta+\gamma_2+\gamma_3-\gamma_2\gamma_3}{1+\delta}$

$y_3^* = \delta(1 - \frac{\delta+\gamma_2+\gamma_3-\gamma_2\gamma_3}{1+\delta})$

If the previous district coopted is district 2,

$x_3^* = \frac{\delta+\gamma_1+\gamma_3-\gamma_1\gamma_3}{1+\delta}$

$y_3^* = \delta(1 - \frac{\delta+\gamma_1+\gamma_3-\gamma_1\gamma_3}{1+\delta})$

Two Previous Elites Coopted

$U_D(accept) = x_1 x_2 x_3$

$U_D(reject) = \delta(1-y_3)x_1 x_2 + (1-\delta)x_1 x_2$

$U_E(accept) = y_3 x_1 x_2$

$U_E(reject) = \delta(1-x_3)x_1 x_2 + (1-\delta)(0)$

$x_3^* = 1$

$y_3^* = 0$

The third elite is superfluous, so she gets no portion of the regime benefit.

Second Bargaining Position

One previous elite coopted (district 1)

If the bargain with district 2 succeeds, the dictator will coopt district 3 with $x_3 = 1$ according to the third bargaining position above

If the bargain with district 2 is unsuccessful, the dictator will coopt district 3 with $x_3 = \frac{\delta+\gamma_2+\gamma_3-\gamma_2\gamma_3}{1+\delta}$

$U_D(accept) = x_1x_2x_3 = x_1x_2(1)$ as $x_3 = 1$

$U_D(reject) = \delta(1-y_2)x_1(1) + (1+\delta)x_1(\frac{\delta+\gamma_2+\gamma_3-\gamma_2\gamma_3}{1+\delta})$ as the regime benefit is still assured because the dictator coopts districts 1 and 3

$U_E(accept) = y_2x_1$

$U_E(reject) = \delta(1-x_2)x_1 + (1-\delta)(0)$

$x_2^* = \frac{2\delta+\delta^2+\gamma_2+\gamma_3-\gamma_2\gamma_3}{(1+\delta)^2}$

$y_2^* = \delta(1 - \frac{2\delta+\delta^2+\gamma_2+\gamma_3-\gamma_2\gamma_3}{(1+\delta)^2})$

No prior elite coopted

If the bargain with district 2 is successful, $x_3 = \frac{\delta+\gamma_1+\gamma_3-\gamma_1\gamma_3}{1+\delta}$

If the bargain fails, the dictator will coopt district 3 where $x_3 = \frac{(\gamma_1\gamma_2+\gamma_1\gamma_3+\gamma_2\gamma_3-2\gamma_1\gamma_2\gamma_3)+\delta(\gamma_1+\gamma_2-\gamma_1\gamma_2)}{(1+\delta)(\gamma_1+\gamma_2-\gamma_1\gamma_2)}$ but the regime benefit will only be achieved (and split) is he wins district 1 or 2 via mass election

$U_D(accept) = x_2(\frac{\delta+\gamma_1+\gamma_3-\gamma_1\gamma_3}{1+\delta})$

$U_D(reject) = \delta(1-y_2)(\frac{\delta+\gamma_1+\gamma_3-\gamma_1\gamma_3}{1+\delta}) + (1-\delta)(\frac{(\gamma_1\gamma_2+\gamma_1\gamma_3+\gamma_2\gamma_3-2\gamma_1\gamma_2\gamma_3)+\delta(\gamma_1+\gamma_2-\gamma_1\gamma_2)}{(1+\delta)(\gamma_1+\gamma_2-\gamma_1\gamma_2)})(\gamma_1+\gamma_2-\gamma_1\gamma_2)$

$U_E(accept) = y_2$

$U_E(reject) = \delta(1-x_2) + (1-\delta)(0)$

$x_2^* = \frac{\gamma_1\gamma_2+\gamma_1\gamma_3+\gamma_2\gamma_3-2\gamma_1\gamma_2\gamma_3+\delta(\delta+\gamma_1+\gamma_2-\gamma_1\gamma_2+\gamma_1+\gamma_3-\gamma_1\gamma_3)}{(1+\delta)(\delta+\gamma_1+\gamma_3-\gamma_1\gamma_3)}$

$y_2^* = \delta(1 - \frac{\gamma_1\gamma_2+\gamma_1\gamma_3+\gamma_2\gamma_3-2\gamma_1\gamma_2\gamma_3+\delta(\delta+\gamma_1+\gamma_2-\gamma_1\gamma_2+\gamma_1+\gamma_3-\gamma_1\gamma_3)}{(1+\delta)(\delta+\gamma_1+\gamma_3-\gamma_1\gamma_3)})$

First Bargaining Position

If the dictator makes a successful bargain with the first elite, $x_3 = 1$ and $x_2 = \frac{2\delta+\delta^2+\gamma_2+\gamma_3-\gamma_2\gamma_3}{(1+\delta)^2}$ all three elites will be coopted

If the bargain with the first elite fails, the dictator will coopt the second elite at $x_2 = \frac{\gamma_1\gamma_2+\gamma_1\gamma_3+\gamma_2\gamma_3-2\gamma_1\gamma_2\gamma_3+\delta(\delta+\gamma_1+\gamma_2-\gamma_1\gamma_2+\gamma_1+\gamma_3-\gamma_1\gamma_3)}{(1+\delta)(\delta+\gamma_1+\gamma_3-\gamma_1\gamma_3)}$ and the third elite at $x_3 = \frac{\delta+\gamma_1+\gamma_3-\gamma_1\gamma_3}{1+\delta}$

$U_D(accept) = x_1x_2x_3 = x_1(\frac{2\delta+\delta^2+\gamma_2+\gamma_3-\gamma_2\gamma_3}{(1+\delta)^2})(1)$

$U_D(reject) = \delta(1-y_1)(\frac{2\delta+\delta^2+\gamma_2+\gamma_3-\gamma_2\gamma_3}{(1+\delta)^2})(1) +$

$(1-\delta)(\frac{\gamma_1\gamma_2+\gamma_1\gamma_3+\gamma_2\gamma_3-2\gamma_1\gamma_2\gamma_3+\delta(\delta+\gamma_1+\gamma_2-\gamma_1\gamma_2+\gamma_1+\gamma_3-\gamma_1\gamma_3)}{(1+\delta)(\delta+\gamma_1+\gamma_3-\gamma_1\gamma_3)})(\frac{\delta+\gamma_1+\gamma_3-\gamma_1\gamma_3}{1+\delta})$

$U_E(accept) = y_1$

$U_E(reject) = \delta(1-x_1) + (1-\delta)(0)$

$x_1^* = \frac{\gamma_1\gamma_2+\gamma_1\gamma_3+\gamma_2\gamma_3-2\gamma_1\gamma_2\gamma_3+\delta(3\delta+\delta^2+\gamma_1+\gamma_2-\gamma_1\gamma_2+\gamma_1+\gamma_3+\gamma_1\gamma_3+\gamma_2+\gamma_3-\gamma_2\gamma_3)}{(1+\delta)(2\delta+\delta^2+\gamma_2+\gamma_3-\gamma_2\gamma_3)}$

$y_1^* = \delta(1-\frac{\gamma_1\gamma_2+\gamma_1\gamma_3+\gamma_2\gamma_3-2\gamma_1\gamma_2\gamma_3+\delta(3\delta+\delta^2+\gamma_1+\gamma_2-\gamma_1\gamma_2+\gamma_1+\gamma_3+\gamma_1\gamma_3+\gamma_2+\gamma_3-\gamma_2\gamma_3)}{(1+\delta)(2\delta+\delta^2+\gamma_2+\gamma_3-\gamma_2\gamma_3)})$

The dictator's overall utility $U_D(x_1, x_2, x_3) = x_1x_2x_3$

$= (\frac{\gamma_1\gamma_2+\gamma_1\gamma_3+\gamma_2\gamma_3-2\gamma_1\gamma_2\gamma_3+\delta(3\delta+\delta^2+\gamma_1+\gamma_2-\gamma_1\gamma_2+\gamma_1+\gamma_3+\gamma_1\gamma_3+\gamma_2+\gamma_3-\gamma_2\gamma_3)}{(1+\delta)(2\delta+\delta^2+\gamma_2+\gamma_3-\gamma_2\gamma_3)})(\frac{2\delta+\delta^2+\gamma_2+\gamma_3-\gamma_2\gamma_3}{(1-\delta)^2})(1)$

$= \frac{\gamma_1\gamma_2+\gamma_1\gamma_3+\gamma_2\gamma_3-2\gamma_1\gamma_2\gamma_3+\delta(3\delta+\delta^2+\gamma_1+\gamma_2-\gamma_1\gamma_2+\gamma_1+\gamma_3+\gamma_1\gamma_3+\gamma_2+\gamma_3-\gamma_2\gamma_3)}{(1-\delta)^2(1+\delta)}$

Investment

$\gamma_1 = \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}$

$\gamma_2 = \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}$

$\gamma_3 = \frac{\alpha_3+\beta_3+\nu_3}{2\beta_3}$

The dictator's true electoral outside option (winning two of three districts with no elite cooptation) is $\gamma_1\gamma_2 + \gamma_1\gamma_3 + \gamma_2\gamma_3 - 2\gamma_1\gamma_2\gamma_3$

Substituting in the district win probabilities,

$$\frac{\beta_1\beta_2(\alpha_3+\nu_3)+\beta_1\beta_3(\alpha_2+\nu_2)+\beta_2\beta_3(\alpha_1+\nu_1)+2\beta_1\beta_2\beta_3-\alpha_1(\nu_2\nu_3+\alpha_2\nu_3+\alpha_3\nu_2)-\alpha_2(\alpha_3\nu_1+\nu_1\nu_3)-\alpha_3(\nu_1\nu_2)-\alpha_1\alpha_2\alpha_3-\nu_1\nu_2\nu_3}{4\beta_1\beta_2\beta_3}$$

The other portion of the numerator from the dictator's utility combines the probabilities of winning districts 1 and/or 2, 2 and/or 3, and 1 and/or 3 (scaled by delta):

$\delta(3\delta + \delta^2 + \gamma_1 + \gamma_2 - \gamma_1\gamma_2 + \gamma_1 + \gamma_3 + \gamma_1\gamma_3 + \gamma_2 + \gamma_3 - \gamma_2\gamma_3)$

Substituting in the district win probabilities,

$$\frac{\delta(3\delta+\delta^2-\beta_1\alpha_2\alpha_3-\alpha_1\alpha_2\beta_3-\beta_1\alpha_2\nu_3-\nu_1\alpha_2\beta_3-\alpha_1\beta_2\alpha_3-\alpha_1\beta_2\nu_3-\nu_1\beta_2\alpha_3-\nu_1\beta_2\nu_3-\beta_1\nu_2\alpha_3-\alpha_1\nu_2\beta_3-\beta_1\nu_2\nu_3-\nu_1\nu_2\beta_3+2\beta_1\alpha_2\beta_3+2\beta_1\beta_2\alpha_3+2}{4\beta_1\beta_2\beta_3}$$

Full equilibrium utility function with three heterogeneous districts:

$$(\delta^3(4\beta_1\beta_2\beta_3) - \delta^2(12\beta_1\beta_2\beta_3) + \delta(\beta_1\alpha_2\alpha_3 + \alpha_1\alpha_2\beta_3 - 2\beta_1\alpha_2\beta_3 + \beta_1\alpha_2\nu_3 + \nu_1\alpha_2\beta_3 + \alpha_1\beta_2\alpha_3 - 2\beta_1\beta_2\alpha_3 - 2\alpha_1\beta_2\beta_3 - 9\beta_1\beta_2\beta_3 + \alpha_1\beta_2\nu_3 - 2\beta_1\beta_2\nu_3 + \nu_1\beta_2\alpha_3 - 2\nu_1\beta_2\beta_3 + \nu_1\beta_2\nu_3 + \beta_1\nu_2\alpha_3 + \alpha_1\nu_2\beta_3 - 2\beta_1\nu_2\beta_3 + \beta_1\nu_2\nu_3 + \nu_1\nu_2\beta_3) + \alpha_1\alpha_2\alpha_3 + \alpha_1\alpha_2\nu_3 + \nu_1\alpha_2\alpha_3 + \nu_1\alpha_2\nu_3 - \beta_1\beta_2\alpha_3 - \alpha_1\beta_2\beta_3 - 2\beta_1\beta_2\beta_3 - \beta_1\beta_2\nu_3 - \nu_1\beta_2\beta_3 + \alpha_1\nu_2\alpha_3 - \beta_1\nu_2\beta_3 + \alpha_1\nu_2\nu_3 + \nu_1\nu_2\alpha_3 + \nu_1\nu_2\nu_3)(\frac{1}{4\beta_1\beta_2\beta_3(1+\delta)(\delta-1)^2}) - c(\nu_1, \nu_2, \nu_3)$$

Weak Local Elites

Weak Elites

Elite will not deliver district with certainty, but with probability η. With more than one district, I will allow the likelihood that an elite can deliver her district to vary across districts. There are possible versions of this: (1) when the dictator and elite come to a cooptation agreement, the elite is paid regardless of her delivery of the district but only if the dictator wins. In other words, if the dictator loses, they both get nothing, if the dictator wins, even if through her own popularity because the elite failed to control the district, the agreement stands and the elite gets her payout. (2) Upon a successful bargain between the dictator and elite, the elite only gets paid if she delivers the district. If the dictator loses, they both get nothing; if the elite fails to deliver the district but the dictator wins anyway through her own popularity, the dictator does not have to pay the elite the agreed upon cooptation price.

Single District

Dictator's outside option is γ, which in this case is the probability he wins the single district without the elite (no investment for now): $\frac{\alpha+\beta}{2\beta}$

Every period the elite proposes x to the dictator and keeps $1-x$ and accepts the dictators proposal if $y \geq y'$. Every period the dictator proposes y, keeping $1-y$ for himself, and accepts the elite proposal if and only if $x \geq x'$. Upon rejection, bargaining continues with probability δ

Version 1: Unconditional Payment to the Elite

Continuation Value (no investment)

$U_D(accept) = \eta x + (1-\eta)\gamma x$

$U_D(reject) = \delta(\eta(1-y) + (1-\eta)\gamma(1-y)) + (1-\delta)\gamma$

Set indifferent

$x = \frac{-\gamma+\delta y\gamma+\delta\eta(-1+y+\gamma-y\gamma)}{\eta(\gamma-1)-\gamma}$

$U_E(accept) = \eta y + (1-\eta)\gamma y$

$U_E(reject) = \delta(\eta(1-x) + (1-\eta)\gamma(1-x)) + (1-\delta)0$

set indifferent

$y = \delta(1-x)$

plug and solve

$x^* = \frac{\gamma+\delta(\eta+\gamma-\eta\gamma)}{(1+\delta)(\eta+\gamma-\eta\gamma)}$

$y^* = \frac{\delta\eta(1-\gamma)}{(1+\delta)(\eta+\gamma-\eta\gamma)}$

$U_D(x^*) = P(win)x^* + P(\neg win)0 = x^*(\eta+\gamma-\eta\gamma)$ as the likelihood that the dictator wins the district is now a function of the elite's ability to deliver and his own popularity

$U_D = \frac{\gamma+\delta(\eta+\gamma-\eta\gamma)}{1+\delta}$

Surplus functional form: the dictator's outside option plus the portion of the mutual surplus that the dictator keeps. How to define the mutual surplus? Before, I defined it as $1-\gamma$ because the likelihood the dictator wins with elite support was 1.

$U_D = \gamma + \frac{\delta}{1+\delta}((\eta + \gamma - \eta\gamma) - \gamma)$

$= \gamma + \frac{\delta}{1+\delta}(\eta(1-\gamma))$

note that this portion (the dictator's share) $\frac{\delta}{1+\delta}$ is the same as the certain elite version with a single district

Note the dictator is still better off coopting the elite and sharing the spoils than going it alone for all $\gamma < 1$ and $\eta > 0$

FULL COMP STATS

Dictator's portion

$x^* = \frac{\gamma+\delta(\eta+\gamma-\eta\gamma)}{(1+\delta)(\eta+\gamma-\eta\gamma)}$

$\frac{\partial}{\partial\eta} = \frac{\gamma(\gamma-1)}{(1+\delta)(\eta+\gamma-\eta\gamma)^2} < 0$ dictator's share decreasing in η

$\frac{\partial}{\partial\gamma} = \frac{\eta}{(1+\delta)(\eta+\gamma-\eta\gamma)^2} > 0$ dictator's share increasing in his outside option likelihood of winning

Dictator's utility

$\gamma + \frac{\delta}{1+\delta}(\eta(1-\gamma))$

$\frac{\partial}{\partial\eta} = \frac{\delta(1-\gamma)}{1+\delta} > 0$ increasing in η

$\frac{\partial}{\partial\gamma} = 1 - \frac{\delta\eta}{1+\delta} > 0$ increasing in γ

Elite's portion $(1 - x^*)$

$= \frac{\eta(\gamma-1)}{(1+\delta)(\eta(\gamma-1)-\gamma)}$

increasing in η

decreasing in γ

Version 2: Conditional Payment

If the elite doesn't deliver the district, they get nothing (even if the dictator wins)

Continuation values:

$U_D(accept) = \eta x + (1-\eta)\gamma + (1-\eta)(1-\gamma)0$

$U_D(reject) = \delta(\eta(1-y) + (1-\eta)\gamma) + (1-\delta)\gamma$

Set indifferent

$x = \delta(1-y) + (1-\delta)\gamma$

$U_E(accept) = \eta y + (1-\eta)0$

$U_E(reject) = \delta(\eta(1-x) + (1-\eta)0) + (1-\delta)0$

set indifferent

$y = \delta(1-x)$

$x^* = \frac{\delta+\gamma}{1+\delta}$ This is the same as the elite with perfect control version

$U_D = \eta x^* + (1-\eta)\gamma + (1-\eta)(1-\gamma)0$

If the elite delivers, they share. If the elite doesn't but the dictator wins through her own popularity, she keeps the whole regime benefit

$U_D = \frac{\gamma+\delta(\eta+\gamma-\eta\gamma)}{1+\delta}$

This ends up being the same as the committed version.... is this because of how the bargain incorporates the elite uncertainty?

The result of the bargain itself x^* is greater with unconditional payment to the elite (the dictator gets more)

Another Version: Conditional on Dictator's Draw

Elite delivers district (according to bargain), but if the dictator's election was sufficiently good such that he would have won anyway (without elite's support), he doesn't have to pay the elite

Continuation Values

With probability γ, the dictator wins the district and doesn't have to share anything with the elite. With prob $1-\gamma$ he needs the elite's support, who delivers with prob η. Bargaining continues with prob δ

$U_D(accept) = \gamma + (1-\gamma)(\eta x + (1-\eta)(0))$

$U_D(reject) = \delta(\gamma + (1-\gamma)(\eta(1-y) + (1-\eta)0)) + (1-\delta)(\gamma + (1-\gamma)0)$

set indifferent

$x = \delta(1-y)$

$U_E(accept) = \gamma(0) + (1-\gamma)(\eta y + (1-\eta)0)$

$U_E(reject) = \delta(\gamma(0) + (1-\gamma)(\eta(1-x) + (1-\eta)(0))$

set indifferent

$y = \delta(1-x)$

$x^* = \frac{\delta}{1+\delta}$

$y^* = \frac{\delta}{1+\delta}$

$U_D = \gamma + (1-\gamma)(\eta x^* + (1-\eta)(0)) = \gamma + \frac{\delta}{1+\delta}(\eta(1-\gamma))$

this is the same as the unconditional weak elite

Investment

Optimal investment for each version

Recall optimal investment for a single district without weak elite (they can deliver the district with certainty) is $\nu^* = c'^{-1}(\frac{1}{2\beta(1+\delta)})$

Where $\nu^* = (\frac{1}{2\beta(1+\delta)}$ if $c(\nu) = \frac{\nu^2}{2}$

Version 1 (Unconditional payment to elite)

$\max_\nu \gamma + \frac{\delta}{1+\delta}(\eta(1-\gamma)) - c(\nu)$

$\gamma = \frac{\alpha+\beta+\nu}{2\beta}$

The assumption about investment here is that it only affects the dictator's outside option, it does not make it "easier" for the elite to deliver the district

FOC: $\frac{1+\delta-\delta\eta}{2\beta(1+\delta} = c'(\nu)$

$\nu^* = c'^{-1}(\frac{1+\delta-\delta\eta}{2\beta(1+\delta)})$

If we set the functional form $c(\nu) = \frac{\nu^2}{2}$

$\nu^* = \frac{1+\delta-\delta\eta}{2\beta(1+\delta)}$

Decreasing in η, δ, β

This level of investment is greater than certain elite investment by $\delta(1-\eta)$

Overall dictator's utility (with optimal investment)

$\gamma = \frac{\alpha+\beta+\nu^*}{2\beta}$ (with cost $c(\nu^*)$)

$\frac{\partial}{\partial\eta} = (d(2b^2(1+d) - 2b(1+d)(1+a+d) + dh))/(4b^2(1+d)^2)$ Not always positive or negative, depends on parameters

increasing in η if $-2\beta(1+\delta)(1+\alpha-\beta+\delta)+\delta\eta > 0$

(from pictures: increasing in η when α is negative, decreasing in η when α is positive, $\alpha \to \beta$, and δ is closer to 1....so really increasing in η for most of the parameter space)

Version 2 (Conditional payment to elite)

Same as V1

Version 3 Conditional on Dictator's Draw

Same as V1

Two of Two Districts

Let the elites deliver their districts with probability η_1 and η_2 where they could be equivalent (cases: $\eta_1 = \eta_2, \eta_1 > \eta_2, \eta_1 < \eta_2$)

Version 1: Unconditional Payment

The dictator will share spoils with the elite(s) regardless of whether they deliver the district

Second Bargaining Position

Subgame: First elite has not been coopted. Regime benefit only achievable if dictator wins district 1 election alone

If the dictator doesn't coopt either elite, $P(win) = \gamma_1\gamma_2$

Continuation Values:

$U_D(accept) = \gamma_1\eta_2 x + \gamma_1\gamma_2(1-\eta_2)x$

$U_D(reject) = (1-\delta)\gamma_1\gamma_2 + \delta(\eta_2\gamma_1(1-y) + (1-\eta_2)\gamma_1\gamma_2(1-y))$

set indifferent

$x(\gamma_1\eta_2 + \gamma_1\gamma_2(1-\eta_2)) = (1-\delta)\gamma_1\gamma_2 + \delta(\eta_2\gamma_1(1-y) + (1-\eta_2)\gamma_1\gamma_2(1-y))$

$x = \frac{y(\delta\eta_2+\delta\gamma_2-\delta\eta_2\gamma_2)}{\eta_2(\gamma_2-1)-\gamma_2} + \frac{-\delta\eta_2-\gamma_2+\delta\eta_2\gamma_2}{\eta_2(\gamma_2-1)-\gamma_2}$

$U_E(accept) = \eta_2\gamma_1 y + (1-\eta_2)\gamma_1\gamma_2 y$

$U_E(reject) = (1-\delta)(0) + \delta(\gamma_1\eta_2(1-x) + (1-\eta_2)\gamma_1\gamma_2(1-x))$

set indifferent

$y(\eta_2\gamma_1 + (1-\eta_2)\gamma_1\gamma_2) = \delta(\gamma_1\eta_2(1-x) + (1-\eta_2)\gamma_1\gamma_2(1-x))$

$y = \delta(1-x)$

solve

$x = \frac{\delta(1-x)*(\delta\eta_2+\delta\gamma_2-\delta\eta_2\gamma_2)}{\eta_2(\gamma_2-1)-\gamma_2} + \frac{-\delta\eta_2-\gamma_2+\delta\eta_2\gamma_2}{\eta_2(\gamma_2-1)-\gamma_2}$

$x^* = \frac{\gamma_2+\delta(\eta_2+\gamma_2-\eta_2\gamma_2)}{(1+\delta)(\eta_2(1-\gamma_2)+\gamma_2)}$

$= \frac{\delta+\frac{\gamma_2}{\eta_2+\gamma_2-\eta_2\gamma_2}}{1+\delta}$

Other Subgame: first elite has been coopted and the dictator must (unconditional payment!) pay the first elite, leaving him x_1 left to bargain with

Continuation values

$U_D(accept) = \eta_1\eta_2 x_1 x_2 + \eta_2(1-\eta_1)(\gamma_1)x_1 x_2 + \eta_1(1-\eta_2)\gamma_2 x_1 x_2 + \gamma_1\gamma_2(1-\eta_1)(1-\eta_2)x_1 x_2$

$U_D(reject) = (1-\delta)(\eta_1\gamma_2 x_1 + \gamma_1\gamma_2(1-\eta_1)x_1) + \delta((1-y_2)x_1(\gamma_1\gamma_2(1-\eta_1)(1-\eta_2) + \eta_1\eta_2 + \eta_1(1-\eta_2)\gamma_2 + \eta_2(1-\eta_1)\gamma_1))$

set indifferent

$x_2 = (-dH - p + dHp + dHY + dpY - dHpY)/(-H - p + Hp)$

$U_E(accept) = y_2 x_1(\eta_1\eta_2 + \eta_2(1-\eta_1)(\gamma_1) + \eta_1(1-\eta_2)\gamma_2 + \gamma_1\gamma_2(1-\eta_1)(1-\eta_2))$

$U_E(reject) = (1-\delta)0 + \delta(1-x_2)x_1((\eta_1\eta_2 + \eta_2(1-\eta_1)(\gamma_1) + \eta_1(1-\eta_2)\gamma_2 + \gamma_1\gamma_2(1-\eta_1)(1-\eta_2))$

$y_2 = \delta(1-x_2)$

Solve

$x_2 = \frac{\delta + \frac{\gamma_2}{\eta_2+\gamma_2-\eta_2\gamma_2}}{1+\delta}$

Confirm that, even if the bargain with the first position fails, the dictator will come to an agreement with the second elite

$U_D(bargain) = x^*(\gamma_1\gamma_2(1-\eta_2) + \gamma_1\eta_2)$

$U_D(alone) = \gamma_1\gamma_2 \ \delta\eta_2(1-\gamma_2) > 0$

True as $\gamma_2 < 1$

First Bargaining Position

The dictator anticipates bargaining with the next person for x_2 portion of the remainder

$U_D(accept) = x_1 x_2(\eta_1\eta_2 + \gamma_1\gamma_2(1-\eta_1)(1-\eta_2) + \eta_1(1-\eta_2)\gamma_2 + \eta_2(1-\eta_1)\gamma_1)$

$U_D(reject) = \delta(x_2(1-y_1)(\eta_1\eta_2 + \gamma_1\gamma_2(1-\eta_1)(1-\eta_2) + \eta_1(1-\eta_2)\gamma_2 + \eta_2(1-\eta_1)\gamma_1)) + (1-\delta)x_2'(\eta_2\gamma_1 + \gamma_1\gamma_2(1-\eta_2))$

$U_E(accept) = y_1 x_2(\eta_1\eta_2 + \gamma_1\gamma_2(1-\eta_1)(1-\eta_2) + \eta_1(1-\eta_2)\gamma_2 + \eta_2(1-\eta_1)\gamma_1)$

$U_E(reject) = (1-\delta)0 + \delta(x_2(1-x_1)(\eta_1\eta_2 + \gamma_1\gamma_2(1-\eta_1)(1-\eta_2) + \eta_1(1-\eta_2)\gamma_2 + \eta_2(1-\eta_1)\gamma_1)$

$y_1 = \delta(1-x_1)$

$x_1^* = \frac{\delta + \frac{\gamma_1}{\eta_1+\gamma_1-\eta_1\gamma_1}}{1+\delta}$

Dictator's full utility:

$x_1^* x_2^*(\eta_1\eta_2 + \gamma_1\gamma_2(1-\eta_1)(1-\eta_2) + \eta_1(1-\eta_2)\gamma_2 + \eta_2(1-\eta_1)\gamma_1)$

$= ((-dh - o + d(-1+h)o)(-dH - p + d(-1+H)p))/(1+d)^2$

$= \frac{\gamma_1\gamma_2 + \delta(\gamma_2(\eta_1 + (1-\eta_1)\gamma_1) + \gamma_1(\eta_2 + (1-\eta_2)\gamma_2) + \delta^2(\eta_1(\gamma_1-1)-\gamma_1)(\eta_2(\gamma_2-1)-\gamma_2))}{(1+\delta)^2}$

COMP STATS

Dictator's Share(s)

$x_1 = \frac{\delta + \frac{\gamma_1}{\eta_1 + \gamma_1 - \eta_1 \gamma_1}}{1+\delta}$

decreasing in η_1

increasing in γ_1

$x_2 = \frac{\delta + \frac{\gamma_2}{\eta_2 + \gamma_2 - \eta_2 \gamma_2}}{1+\delta}$

decreasing in η_2

increasing in γ_2

Elites' Shares

$(1 - x_1)$

Increasing in η_1

decreasing in γ_1

$(1 - x_2)$

Increasing in η_2

decreasing in γ_2

Dictator overall utility $\frac{\gamma_1\gamma_2+\delta(\gamma_2(\eta_1+(1-\eta_1)\gamma_1)+\gamma_1(\eta_2+(1-\eta_2)\gamma_2)+\delta^2(\eta_1(\gamma_1-1)-\gamma_1)(\eta_2(\gamma_2-1)-\gamma_2))}{(1+\delta)^2}$

$\frac{\partial}{\partial\eta_1} = \frac{\delta(\gamma_1-1)(\delta\eta_2(\gamma_2-1)-\gamma_2-\delta\gamma_2)}{(1+\delta)^2} > 0$ increasing in η_1

$\frac{\partial}{\partial\eta_2} = \frac{(-\delta\eta_1-\gamma_1+\delta(\eta_1-1)\gamma_1)(-\delta+\delta\gamma_2)}{(1+\delta)^2} > 0$ increasing in η_2

$\frac{\partial}{\partial\gamma_1} > 0$

$\frac{\partial}{\partial\gamma_2} > 0$

Version 2: Conditional Payment

Elite only gets paid if they deliver their district

Second bargaining position

Subgame first elite not coopted

$U_D(accept) = \gamma_1(\eta_2 x_2 + (1-\eta_2)\gamma_2)$

$U_D(reject) = \delta(\gamma_1\eta_2(1-y_2) + (1-\eta_2)\gamma_1\gamma_2) + (1-\delta)\gamma_1\gamma_2$

set indifferent

$x_2 = \delta(1-y_2) + \gamma_2(1-\delta)$

$U_E(accept) = \gamma_1\eta_2 y_2$

$U_E(reject) = \delta\gamma_1\eta_2(1-x_2)$

$y_2 = \delta(1-x_2)$

$x_2' = \frac{\delta+\gamma_2}{1+\delta}$

Subgame first elite coopted

$U_D(accept) = x_1x_2\eta_1\eta_2 + \eta_1(1-\eta_2)\gamma_2x_1 + \eta_2(1-\eta_1)\gamma_1x_2 + (1-\eta_1)(1-\eta_2)\gamma_1\gamma_2$

$U_D(reject) = \delta(x_1\eta_1\eta_2(1-y_2) + \eta_1(1-\eta_2)\gamma_2x_1 + \eta_2(1-\eta_1)\gamma_1(1-y_2) + (1-\eta_1)(1-\eta_2)\gamma_1\gamma_2) +$
$(1-\delta)(\gamma_2\eta_1x_1 + \gamma_2\gamma_1(1-\eta_1))$

$U_E(accept) = x_1y_2\eta_1\eta_2 + y_2\eta_2(1-\eta_1)\gamma_1$

$U_E(reject) = \delta(x_1(1-x_2)\eta_1\eta_2 + (1-x_2)\eta_2(1-\eta_1)\gamma_1)$

$y_2 = \delta(1-x_2)$

$x_2 = \frac{\delta+\gamma_2}{1+\delta}$

First bargaining position

$U_D(accept) = x_1x_2\eta_1\eta_2 + \eta_1(1-\eta_2)\gamma_2x_1 + \eta_2(1-\eta_1)\gamma_1x_2 + (1-\eta_1)(1-\eta_2)\gamma_1\gamma_2$

$U_D(reject) = \delta(\eta_1\eta_2 x_2(1-y_1) + \eta_1(1-\eta_2)\gamma_2(1-y_1) + \eta_2(1-\eta_1)\gamma_1 x_2 + (1-\eta_1)(1-\eta_2)\gamma_1\gamma_2) + (1-\delta)(\gamma_1\eta_2 x_2' + \gamma_1(1-\eta_2)\gamma_2)$

$U_E(accept) = x_2 y_2 \eta_1\eta_2 + \eta_1(1-\eta_2)\gamma_2 y_1$

$U_E(reject) = \delta(x_2(1-x_1)\eta_1\eta_2 + (1-x_1)\eta_1(1-\eta_2)\gamma_2)$

$y = \delta(1-x_1)$

$x_1 = \frac{\delta+\gamma_1}{1+\delta}$

$U_D = \eta_1\eta_2 x_1^* x_2^* + \eta_1(1-\eta_2)\gamma_2 x_1 + \eta_2(1-\eta_1)\gamma_1 x_2 + (1-\eta_1)(1-\eta_2)\gamma_1\gamma_2$

$= \frac{\gamma_1\gamma_2+\delta(\eta_1\gamma_2+\eta_2\gamma_1+\gamma_1\gamma_2(2-\eta_1-\eta_2))+\delta^2(\eta_1\eta_2+\gamma_2(\eta_1-\eta_1\eta_2)+\gamma_1(\eta_2-\eta_1\eta_2)+\gamma_1\gamma_2(\eta_1-1)(\eta_2-1)}{(1+\delta)^2}$

SAME AS VERSION 1

Version 3 Conditional on Dictator Payment

2nd bargaining position

subgame: first elite not coopted

$U_D(accept) = \gamma_1\gamma_2 + \gamma_1(1-\gamma_2)\eta_2 x_2$

$U_D(reject) = \delta(\gamma_1\gamma_2 + \gamma_1(1-\gamma_2)\eta_2(1-y_2)) + (1-\delta)\gamma_1\gamma_2$

$U_E(accept) = (1-\gamma_2)\gamma_1\eta_2 y_2$

$U_E(reject) = \delta((1-\gamma_2)\gamma_1\eta_2(1-x_2))$

$y_2 = \delta(1-x_2)$

$x_2' = \frac{\delta}{1+\delta}$

Subgame: first elite coopted

$U_D(accept) = \gamma_1\gamma_2 + \gamma_1(1-\gamma_2)\eta_2 x_2 + \gamma_2(1-\gamma_1)\eta_1 x_1 + (1-\gamma_1)(1-\gamma_2)\eta_1\eta_2 x_1 x_2$

$U_D(reject) = \delta(\gamma_1\gamma_2 + \gamma_1(1-\gamma_2)\eta_2(1-y_2) + \gamma_2(1-\gamma_1)\eta_1 x_1 + (1-\gamma_1)(1-\gamma_2)\eta_1\eta_2 x_1(1-y_2)) + (1-\delta)(\gamma_1\gamma_2 + \gamma_2(1-\gamma_1)\eta_1 x_1)$

$U_E(accept) = (1-\gamma_2)\eta_2\gamma_1 y_2 + (1-\gamma_1)(1-\gamma_2)\eta_1\eta_2 x_1 y_2$

$U_E(reject) = \delta((1-\gamma_2)\eta_2\gamma_1(1-x_2) + (1-\gamma_1)(1-\gamma_2)\eta_1\eta_2 x_1(1-x_2))$

$y_2 = \delta(1-x_2)$

$x_2 = \frac{\delta}{1+\delta}$

First Bargaining Position

$U_D(accept) = \gamma_1\gamma_2 + \gamma_1(1-\gamma_2)\eta_2 x_2 + \gamma_2(1-\gamma_1)\eta_1 x_1 + (1-\gamma_1)(1-\gamma_2)\eta_1\eta_2 x_1 x_2$

$U_D(reject) = \delta(\gamma_1\gamma_2 + \gamma_1(1-\gamma_2)\eta_2 x_2 + \gamma_2(1-\gamma_1)\eta_1(1-y_1) + (1-\gamma_1)(1-\gamma_2)\eta_1\eta_2(1-y_1)x_2) + (1-\delta)(\gamma_1\gamma_2 + \gamma_1(1-\gamma_2)\eta_2 x_2')$

$U_E(accept) = (1-\gamma_1)\eta_1\gamma_2 y_1 + (1-\gamma_1)(1-\gamma_2)\eta_1\eta_2 x_2 y_1$

$U_E(reject) = \delta((1-\gamma_1)\eta_1\gamma_2(1-x_1) + (1-\gamma_1)(1-\gamma_2)\eta_1\eta_2 x_2(1-x_1))$

$y_1 = \delta(1-x_1)$

$x_1 = \frac{\delta}{1+\delta}$

$U_D = \gamma_1\gamma_2 + \gamma_1(1-\gamma_2)\eta_2 x_2^* + \gamma_2(1-\gamma_1)\eta_1 x_1^* + (1-\gamma_1)(1-\gamma_2)\eta_1\eta_2 x_1^* x_2^*$

same as above!

Investment

As with the baseline heterogeneous district version, let $V_1 \sim U[-\beta_1 + \alpha_1, \beta_1 + \alpha_1]$ and $V_2 \sim U[-\beta_2 + \alpha_2, \beta_2 + \alpha_2]$ and when the dictator makes investment ν_i in district i, his expected probability of winning the district without elite support is $\frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}$

The cost of investment $c(\nu_1, \nu_2) = \frac{\nu_1^2}{2} + \frac{\nu_2^2}{2} + \chi\nu_1\nu_2$ where $\chi \geq 0$ is the complementarity of costs from investing in each district.

Objective Function:

$\frac{\gamma_1\gamma_2+\delta(\eta_1\gamma_2+\eta_2\gamma_1+\gamma_1\gamma_2(2-\eta_1-\eta_2))+\delta^2(\eta_1\eta_2+\gamma_2(\eta_1-\eta_1\eta_2)+\gamma_1(\eta_2-\eta_1\eta_2)+\gamma_1\gamma_2(\eta_1-1)(\eta_2-1)}{(1+\delta)^2} - c(\nu_1,\nu_2)$ where $\gamma_1 = \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}$ and $\gamma_2 = \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}$

First Order Conditions:

$\frac{\partial}{\partial\nu_1} = -v-((-1+d(-1+h))(A(1+d-dH)+B(1+d+dH)+(1+d-dH)V))/(4bB(1+d)^2)-VX=0$

$\frac{\partial}{\partial\nu_2} = -(((-1+d(-1+H))(b+a(1+d-dh)+v+d(b+bh+v-hv)))/(4bB(1+d)^2))-V-vX$

Second Partials:

$\frac{\partial^2}{\partial\nu_2\nu_1} = ((-1+d(-1+h))(-1+d(-1+H)))/(4bB(1+d)^2)-X$

$\frac{\partial^2}{\partial\nu_1\nu_2} = ((-1+d(-1+h))(-1+d(-1+H)))/(4bB(1+d)^2)-X$

$\frac{\partial^2}{\partial\nu_1^2} = -1$

$\frac{\partial^2}{\partial\nu_2^2} = -1$

Hessian: $\begin{bmatrix} -1 & \frac{1+\delta(1-\eta_1+1-\eta_2)+\delta^2(\eta_1-1)(\eta_2-1)}{4\beta_1\beta_2(1+\delta)^2}-\chi \\ \frac{1+\delta(1-\eta_1+1-\eta_2)+\delta^2(\eta_1-1)(\eta_2-1)}{4\beta_1\beta_2(1+\delta)^2}-\chi & -1 \end{bmatrix}$

$$\begin{bmatrix} -1 & \frac{1+\delta(1-\eta_1+1-\eta_2)+\delta^2(\eta_1-1)(\eta_2-1)}{4\beta_1\beta_2(1+\delta)^2}-\chi \\ \frac{1+\delta(1-\eta_1+1-\eta_2)+\delta^2(\eta_1-1)(\eta_2-1)}{4\beta_1\beta_2(1+\delta)^2}-\chi & -1 \end{bmatrix} - \lambda I =$$

$$\begin{bmatrix} -1-\lambda & \frac{1+\delta(1-\eta_1+1-\eta_2)+\delta^2(\eta_1-1)(\eta_2-1)}{4\beta_1\beta_2(1+\delta)^2}-\chi \\ \frac{1+\delta(1-\eta_1+1-\eta_2)+\delta^2(\eta_1-1)(\eta_2-1)}{4\beta_1\beta_2(1+\delta)^2}-\chi & -1-\lambda \end{bmatrix}$$

Characteristic polynomial: $(-1-\lambda)^2 - (\frac{1+\delta(1-\eta_1+1-\eta_2)+\delta^2(\eta_1-1)(\eta_2-1)}{4\beta_1\beta_2(1+\delta)^2}-\chi)^2 = 0$

$\lambda = -1 \pm (\frac{1+\delta(1-\eta_1+1-\eta_2)+\delta^2(\eta_1-1)(\eta_2-1)}{4\beta_1\beta_2(1+\delta)^2}-\chi)$

$\lambda = -1 + (\frac{1+\delta(1-\eta_1+1-\eta_2)+\delta^2(\eta_1-1)(\eta_2-1)}{4\beta_1\beta_2(1+\delta)^2}-\chi) < 0$ by definition of parameters

$\lambda = -1 - (\frac{1+\delta(1-\eta_1+1-\eta_2)+\delta^2(\eta_1-1)(\eta_2-1)}{4\beta_1\beta_2(1+\delta)^2}-\chi) < 0$ if χ is sufficiently small

$\chi < 1 + \frac{(-1+\delta(\eta_1-1))(-1+\delta(\eta_2-1))}{4\beta_1\beta_2(1+\delta)^2}$

Need $\beta_1, \beta_2 \geq 0.5$ and $\chi < 1 + \frac{(-1+\delta(\eta_1-1))(-1+\delta(\eta_2-1))}{4\beta_1\beta_2(1+\delta)^2}$ (the condition on βs are actually not quite that strict, but similar to the condition on districts being sufficiently uncertain for the baseline heterogeneous investment comparative statics). If these conditions hold, the hessian is negative definite and the solution is a local max.

Solution:

$\nu_1^* = -\frac{(-1+\delta(\eta_1-1))(\beta_2+\alpha_2(1+\delta-\delta\eta_2)+\nu_2+\delta(\beta_2+\beta_2\eta_2+\nu_2-\eta_2\nu_2))}{4\beta_1\beta_2(1+\delta)^2} - \chi\nu_2$

$\nu_2^* = -\frac{(-1+\delta(\eta_2-1))(\beta_1+\alpha_1(1+\delta-\delta\eta_1)+\nu_1+\delta(\beta_1+\beta_1\eta_1+\nu_1-\eta_1\nu_1)}{4\beta_1\beta_2(1+\delta)^2} - \chi\nu_1$

$\nu_2* = -((A(-1+d(-1+h))(-1+d(-1+H))((-1+d(-1+h))(-1+d(-1+H))-4bB(1+d)^2X)+B(4ab(1+d)^2(-1+d(-1+h))(-1+d(-1+H))-4b^2(1+d)^2(1+d+dh)(-1+d(-1+H))-(1+d-dh)^2(-(1+d)^2+d^2H^2)+4bB(1+d)^2(-1+d(-1+h))(1+d+dH)X))/((1+d-dh)^2(1+d-dH)^2-8bB(1+d)^2(-1+d(-1+h))(-1+d(-1+H))X+16b^2B^2(1+d)^4(-1+X^2)))$

$\nu_1^* = -((a(-1+d(-1+h))(-1+d(-1+H))((-1+d(-1+h))(-1+d(-1+H))-4bB(1+d)^2X)+b(4AB(1+d)^2(-1+d(-1+h))(-1+d(-1+H))-(-(1+d)^2+d^2h^2)(1+d-dH)^2-4B^2(1+d)^2(-1+d(-1+h))(1+d+dH)+4bB(1+d)^2(1+d+dh)(-1+d(-1+H))X))/((1+d-dh)^2(1+d-dH)^2-8bB(1+d)^2(-1+d(-1+h))(-1+d(-1+H))X+16b^2B^2(1+d)^4(-1+X^2)))$

ν_2^* comparative statics

$\frac{\partial\nu_2^*}{\partial\alpha_1}$ positive (when maintaining restrictions on βs and χ

$\frac{\partial\nu_2^*}{\partial\alpha_2}$ can be positive or negative depending on parameters, but negative for most of the parameter space (only positive for really small βs)

$\frac{\partial\nu_2^*}{\partial\eta_1}$ depends on αs and βs

$\frac{\partial\nu_2^*}{\partial\eta_2}$ depends αs: if α_1 is positive, decreasing in η_2. If α_1 is negative, sign depends on βs

$\frac{\partial\nu_2^*}{\partial\beta}$ depends on relative parameters (for both β s)

ν_1^* comparative statics

$\frac{\partial \nu_1^*}{\partial \alpha_1}$ mostly negative

$\frac{\partial \nu_1^*}{\partial \alpha_2}$ positive

$\frac{\partial \nu_1^*}{\partial \eta_1}$ depends on αs and βs

Check corner solutions (when χ is too big)

$(0, 0)$

$\frac{\gamma_1\gamma_2+\delta(\eta_1\gamma_2+\eta_2\gamma_1+\gamma_1\gamma_2(2-\eta_1-\eta_2))+\delta^2(\eta_1\eta_2+\gamma_2(\eta_1-\eta_1\eta_2)+\gamma_1(\eta_2-\eta_1\eta_2)+\gamma_1\gamma_2(\eta_1-1)(\eta_2-1)}{(1+\delta)^2}$ where $\gamma_1 = \frac{\alpha_1+\beta_1}{2\beta_1}$ and $\gamma_2 = \frac{\alpha_2+\beta_2}{2\beta_2}$

utility with no investment: $(((a+b)(1+d)+(-a+b)dh)((A+B)(1+d)+(-A+B)dH))/(4bB(1+d)^2)$

$(\nu_1, 0)$

$\frac{\gamma_1\gamma_2+\delta(\eta_1\gamma_2+\eta_2\gamma_1+\gamma_1\gamma_2(2-\eta_1-\eta_2))+\delta^2(\eta_1\eta_2+\gamma_2(\eta_1-\eta_1\eta_2)+\gamma_1(\eta_2-\eta_1\eta_2)+\gamma_1\gamma_2(\eta_1-1)(\eta_2-1)}{(1+\delta)^2} - c(\nu_1)$ where $\gamma_1 = \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}$ and $\gamma_2 = \frac{\alpha_2+\beta_2}{2\beta_2}$

Objective function:$(1/(4bB(1+d)^2))(((a+b)(1+d)+(-a+b)dh)((A+B)(1+d)+(-A+B)dH)-(-1+d(-1+h))((A+B)(1+d)+(-A+B)dH)v-2bB(1+d)^2v^2)$

$\nu_1 = ((-1-d+dh)(-A-B-Ad-Bd+AdH-BdH))/(4bB(1+d)^2)$ decreasing in η_1, increasing in η_2, increasing in α_2, decreasing in β_1

utility with optimal investment: $(1/(32b^2B^2(1+d)^4))((A+B)(1+d)+(-A+B)dH)(-A(1+d-dh)^2(-1+d(-1+H))+B(-8ab(1+d)^2(-1+d(-1+h))+8b^2(1+d)^2(1+d+dh)+(1+d-dh)^2(1+d+dH)))$

$\nu_1 = \frac{(-1+\delta(\eta_1-1))(-\delta\eta_2-\gamma_2+\delta(-1+\eta_2)\gamma_2)}{2\beta_1(1+\delta)^2}$

$(0, \nu_2)$

$\frac{\gamma_1\gamma_2+\delta(\eta_1\gamma_2+\eta_2\gamma_1+\gamma_1\gamma_2(2-\eta_1-\eta_2))+\delta^2(\eta_1\eta_2+\gamma_2(\eta_1-\eta_1\eta_2)+\gamma_1(\eta_2-\eta_1\eta_2)+\gamma_1\gamma_2(\eta_1-1)(\eta_2-1)}{(1+\delta)^2} - c(\nu_2)$ where $\gamma_1 = \frac{\alpha_1+\beta_1}{2\beta_1}$ and $\gamma_2 = \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}$

Objective function: $(1/(4bB(1+d)^2))(((a+b)(1+d)+(-a+b)dh)((A+B)(1+d)+(-A+B)dH)-((a+b)(1+d)+(-a+b)dh)(-1+d(-1+H))V-2bB(1+d)^2V^2)$

$\nu_2 = -((((a+b)(1+d)+(-a+b)dh)(-1+d(-1+H)))/(4bB(1+d)^2))$

Utility with optimal investment: $(1/(32b^2B^2(1+d)^4))((a+b)(1+d)+(-a+b)dh)(8bB(A+B)(1+d)^3+(-a+b)dh(-1+d(-1+H))^2+8bB(-A+B)d(1+d)^2H+(a+b)(1+d)(1+d-dH)^2)$

No investment is dominated, whether he should invest in district 1 or 2 depends on relative parameters

For simplicity: when $\beta_1 = \beta_2$, invest where less popular; when $\alpha_1 = \alpha_2$ invest where β is lower (investment is more effective)

invest in D1 instead of D2 iff $-(((1+d)((a+A+b+B)(1+d)-(a+A-b+B)dh)+d(-((a+A+b-B)(1+d))+(a+A-b-B)dh)H)((1+d)((a-A+b-B)(1+d)+(-a+A+b+B)dh)+d(-((a-A+b+B)(1+d))+(a-A-b+B)dh)H)) > 0$

Two Districts: One of Two

Version 1 Unconditional Payment

2nd Bargaining Position, first not coopted

$U_D(accept) = \eta_2 x_2 + (1-\eta_2)(\gamma_1+\gamma_2-\gamma_1\gamma_2)x_2$

$U_D(reject) = \delta(\eta_2(1-y_2)+(1-\eta_2)(\gamma_1+\gamma_2-\gamma_1\gamma_2)(1-y_2)) + (1-\delta)(\gamma_1+\gamma_2-\gamma_1\gamma_2)(1)$

$U_E(accept) = \eta_2 y_2 + (1-\eta_2)(\gamma_1+\gamma_2-\gamma_1\gamma_2)y_2$

$U_E(reject) = \delta(\eta_2(1-x_2)+(1-\eta_2)(\gamma_1+\gamma_2-\gamma_1\gamma_2)(1-x_2)$

$y_2 = \delta(1-x_2)$

$x_2^* = \frac{\gamma_1+\gamma_2-\gamma_1\gamma_2+\delta(\gamma_1+\gamma_2-\gamma_1\gamma_2+\eta_2(\gamma_1-1)(\gamma_2-1))}{(1+\delta)(\gamma_1+\gamma_2-\gamma_1\gamma_2+\eta_2(\gamma_1-1)(\gamma_2-1))}$

2nd Bargaining position, first coopted

$U_D(accept) = x_1x_2(\eta_1+\eta_2-\eta_1\eta_2+(1-\eta_1)(1-\eta_2)(\gamma_1+\gamma_2-\gamma_1\gamma_2)$

$U_D(reject) = \delta(x_1(1-y_2)(\eta_1+\eta_2-\eta_1\eta_2+(1-\eta_1)(1-\eta_2)(\gamma_1+\gamma_2-\gamma_1\gamma_2))+(1-\delta)(x_1(\eta_1+(1-\eta_1)(\gamma_1+\gamma_2-\gamma_1\gamma_2))$

$U_E(accept) = x_1 y_2((\eta_1 + \eta_2 - \eta_1\eta_2 + (1-\eta_1)(1-\eta_2)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2))$

$U_E(reject) = \delta(x_1(1-x_2)((\eta_1 + \eta_2 - \eta_1\eta_2 + (1-\eta_1)(1-\eta_2)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2))$

$y_2 = \delta(1-x_2)$

$x_2^* = -\frac{\gamma_1 - \eta_1(-1+\delta(\eta_2-1))(\gamma_1-1)(\gamma_2-1)+\gamma_2-\gamma_1\gamma_2+\delta(\eta_2+\gamma_1-\eta_2\gamma_1+(\eta_2-1)(\gamma_1-1)\gamma_2)}{(1+\delta)(-\gamma_1+\eta_1(1-\eta_2)(\gamma_1-1)(\gamma_2-1)+(\gamma_1-1)\gamma_2+\eta_2(-1+\gamma_1+\gamma_2-\gamma_1\gamma_2))}$

First Bargaining Position

$U_D(accept) = x_1 x_2((\eta_1 + \eta_2 - \eta_1\eta_2 + (1-\eta_1)(1-\eta_2)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2))$

$U_D(reject) = \delta(x_2(1-y_1)(\eta_1 + \eta_2 - \eta_1\eta_2 + (1-\eta_1)(1-\eta_2)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2)) + (1-\delta)(x_2'(\eta_2 + (1-\eta_2)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2))$

$U_E(accept) = x_2 y_1(\eta_1 + \eta_2 - \eta_1\eta_2 + (1-\eta_1)(1-\eta_2)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2)$

$U_E(reject) = \delta(x_2(1-x_1)(\eta_1 + \eta_2 - \eta_1\eta_2 + (1-\eta_1)(1-\eta_2)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2))$

$y_1 = \delta(1-x_1)$

$x_1 = -((-((\gamma_1(\gamma_2-1)-\gamma_2)(\eta_2(\gamma_1-1)+\gamma_1(\gamma_2-1)-\gamma_2))-\delta(\eta_1+\eta_2+2\gamma_1-\eta_1\gamma_1-\eta_2\gamma_1+(\eta_1-2)(\gamma_1-1)\gamma_2)(\gamma_1+\eta_2(\gamma_1-1)(\gamma_2-10+\gamma_2-\gamma_1\gamma_2)+\delta^2(\gamma_1+\eta_2(\gamma_1-1)(\gamma_2-1)+\gamma_2-\gamma_1\gamma_2)(-\gamma_1+\eta_1(\eta_2-1)(\gamma_1-1)(\gamma_2-1)+(\gamma_1-1)\gamma_2+\eta_2(-1+\gamma_1+\gamma_2-\gamma_1\gamma_2))/(1+\delta)(\gamma_1+\eta_2(\gamma_1-1)(\gamma_2-1)+\gamma_2-\gamma_1\gamma_2)(\gamma_1-\eta_1(-1+\delta(\eta_2-1))(\gamma_1-1)(\gamma_2-1)+\gamma_2-\gamma_1\gamma_2+\delta(\eta_2+\gamma_1-\eta_2\gamma_1+(\eta_2-1)(\gamma_1-1)\gamma_2))))$

$U_D = x_1^* x_2^*(\eta_1 + \eta_2 - \eta_1\eta_2 + (1-\eta_1)(1-\eta_2)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2)$

$= \frac{\gamma_1+\gamma_2-\gamma_1\gamma_2+\delta(\eta_1(\gamma_1-1)(\gamma_2-1)+\eta_2(\gamma_1-1)(\gamma_2-1)+2(\gamma_1+\gamma_2-\gamma_1\gamma_2))+\delta^2(\gamma_1+\gamma_2-\gamma_1\gamma_2-\eta_1(\eta_2-1)(\gamma_1-1)(\gamma_2-1)+\eta_2(\gamma_1-1)(\gamma_2-1))}{(1+\delta)^2}$

Confirmed: the dictator's expected utility is better when he coopts both elites as opposed to just the first or just the second

$\Omega + ((o + p - op - ((-1+d)(h + H - hH + (1-h)(1-H)(o+p-op))(-o + h(-1 + d(-1+H))(-1+o)(-1+p) + (-1+o)p - d(o + H(-1+o)(-1+p) + p - op))(d - d^2 + ((o + p - op + d(o + H(-1+o)(-1+p) + p - op))(H + o + (-2+H)Ho + h(-1+H)(-1+d^2(-1+o)(-1+p)) + p - ((-2+H)H(-1+o)+o)p - d^2(o + H(-1+o)(-1+p) + p - op)))/((1+d)(-o + h(-1+H)(-1+o)(-1+p) + (-1+o)p + H(-1+o+p-op))(-o + h(-1+d(-1+$

$H))(-1+o)(-1+p)+(-1+o)p-d(o+H(-1+o)(-1+p)+p-op)))))/((1-d^2)(H+o+(-2+H)Ho+h(-1+H)(-1+d^2(-1+o)(-1+p))+p-((-2+H)H(-1+o)+o)p-d^2(o+H(-1+o)(-1+p)+p-op))))/((h(-1+H)-H)(-1+o)(-1+p)))(1-\Omega)$

Comp Stats

Dictator's Share

x_2

x_1

Increasing in η_1 when γ_1 and γ_2 are sufficiently high (and δ high)

increasing in η_1 when γ_1 and γ_2 are sufficiently high (and δ high) AND $\eta_2 >> \eta_1$

(mostly) increasing in γ_1

Elites' Shares

Dictator's Utility $x_1^* x_2^*(\eta_1+\eta_2-\eta_1\eta_2+(1-\eta_1)(1-\eta_2)(\gamma_1+\gamma_2-\gamma_1\gamma_2)$

$=((-1+d)(-o+h(-1+d(-1+H))(-1+o)(-1+p)+(-1+o)p-d(H+o-Ho+(-1+H)(-1+o)p))(h+H-hH+(1-h)(1-H)(o+p-op))(d-d^2+((o+p-op+d(o+H(-1+o)(-1+p)+p-op))(H+o+(-2+H)Ho+h(-1+H)(-1+d^2(-1+o)(-1+p))+p-((-2+H)H(-1+o)+o)p-d^2(o+H(-1+o)(-1+p)+p-op)))/((1+d)(-o+h(-1+d(-1+H))(-1+o)(-1+p)+(-1+o)p-d(H+o-Ho+(-1+H)(-1+o)p))(-o+h(-1+H)(-1+o)(-1+p)+(-1+o)p+H(-1+o+p-op)))))/((1-d^2)(H+o+(-2+H)Ho+h(-1+H)(-1+d^2(-1+o)(-1+p))+p-((-2+H)H(-1+o)+o)p-d^2(o+H(-1+o)(-1+p)+p-op)))$

Investment

Objective Function

$\frac{\gamma_1+\gamma_2-\gamma_1\gamma_2+\delta(\eta_1(\gamma_1-1)(\gamma_2-1)+\eta_2(\gamma_1-1)(\gamma_2-1)+2(\gamma_1+\gamma_2-\gamma_1\gamma_2))+\delta^2(\gamma_1+\gamma_2-\gamma_1\gamma_2-\eta_1(\eta_2-1)(\gamma_1-1)(\gamma_2-1)+\eta_2(\gamma_1-1)(\gamma_2-1))}{(1+\delta)^2} - c(\nu_1,\nu_2)$

Where $\gamma_1 = \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}$, $\gamma_2 = \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}$ and $c(\nu_1,\nu_2) = \frac{\nu_1^2}{2} + \frac{\nu_2^2}{2} + \chi\nu_1\nu_2$

First Order Conditions

$\frac{\partial}{\partial\nu_1} = -v-((-1+d(-1+h))(-1+d(-1+H))(A-B+V))/(4bB(1+d)^2)-VX = 0$

$\frac{\partial}{\partial \nu_2} = -(((-1+d(-1+h))(-1+d(-1+H))(a-b+v))/(4bB(1+d)^2)) - V - vX = 0$

Second Partials

$\frac{\partial^2}{\partial \nu_1^2} = -1$

$\frac{\partial^2}{\partial \nu_1 \nu_2} = -(((-1+d(-1+h))(-1+d(-1+H)))/(4bB(1+d)^2)) - X$

$\frac{\partial^2}{\partial \nu_2 \nu_1} = -(((-1+d(-1+h))(-1+d(-1+H)))/(4bB(1+d)^2)) - X$

$\frac{\partial^2}{\partial \nu_2^2} = -1$

Hessian:

$$\begin{bmatrix} -1 & -\frac{1+\delta(1-\eta_1+1-\eta_2)+\delta^2(\eta_1-1)(\eta_2-1)}{4\beta_1\beta_2(1+\delta)^2} - \chi \\ -\frac{1+\delta(1-\eta_1+1-\eta_2)+\delta^2(\eta_1-1)(\eta_2-1)}{4\beta_1\beta_2(1+\delta)^2} - \chi & -1 \end{bmatrix}$$

Characteristic polynomial:

$(-1-\lambda)^2 - (-\frac{1+\delta(1-\eta_1+1-\eta_2)+\delta^2(\eta_1-1)(\eta_2-1)}{4\beta_1\beta_2(1+\delta)^2} - \chi)^2$

$\lambda = -1 \pm (\frac{(-1+\delta(\eta_1-1))(-1+\delta(\eta_2-1))}{4\beta_1\beta_2(1+\delta)^2} + \chi)$

Need similar restrictions on χ and βs for negative semi-definite hessian

Negative definite if $\chi < 1 - \frac{(\delta(\eta_1-1)-1)(\delta(\eta_2-1)-1)}{4\beta_1\beta_2(1+\delta)^2}$

$\nu_1 = -((-1+d(-1+h))(-1+d(-1+H))(A-B+V))/(4bB(1+d)^2) - VX$

$\nu_2 = -(((-1+d(-1+h))(-1+d(-1+H))(a-b+v))/(4bB(1+d)^2)) - vX$

Comparative Statics

$\nu_1 = -(((-1+d(-1+h))(-1+d(-1+H))(a(-1+d(-1+h))(-1+d(-1+H))+4abB(1+d)^2X - b(4AB(1+d)^2 - 4B^2(1+d)^2 + (-1+d(-1+h))(-1+d(-1+H)) + 4bB(1+d)^2X)))/(16b^2B^2(1+d)^4(-1+((-1+d(-1+h))(-1+d(-1+H))+4bB(1+d)^2X)^2/(16b^2B^2(1+d)^4))))$

$\frac{\partial}{\partial \alpha_1}$ positive

$\frac{\partial}{\partial \alpha_2}$ negative

$\frac{\partial}{\partial \eta_1}$ negative

$\frac{\partial}{\partial \eta_2}$ negative

$\nu_2 = -(((-1 + d(-1 + h))(-1 + d(-1 + H))(A(-1 + d(-1 + h))(-1 + d(-1 + H)) + 4AbB(1 + d)^2X - B(4ab(1 + d)^2 - 4b^2(1 + d)^2 + (-1 + d(-1 + h))(-1 + d(-1 + H)) + 4bB(1 + d)^2X)))/(16b^2B^2(1 + d)^4(-1 + ((-1 + d(-1 + h))(-1 + d(-1 + H)) + 4bB(1 + d)^2X)^2/(16b^2B^2(1 + d)^4))))$

$\frac{\partial}{\partial \alpha_1}$ negative

$\frac{\partial}{\partial \alpha_2}$ positive

Corner Solutions (when χ is too large)

(0, 0)

$\frac{\gamma_1+\gamma_2-\gamma_1\gamma_2+\delta(\eta_1(\gamma_1-1)(\gamma_2-1)+\eta_2(\gamma_1-1)(\gamma_2-1)+2(\gamma_1+\gamma_2-\gamma_1\gamma_2))+\delta^2(\gamma_1+\gamma_2-\gamma_1\gamma_2-\eta_1(\eta_2-1)(\gamma_1-1)(\gamma_2-1)+\eta_2(\gamma_1-1)(\gamma_2-1))}{(1+\delta)^2} - c(\nu_1, \nu_2)$

Where $\gamma_1 = \frac{\alpha_1+\beta_1}{2\beta_1}$, $\gamma_2 = \frac{\alpha_2+\beta_2}{2\beta_2}$

$(\nu_1, 0)$

$\frac{\gamma_1+\gamma_2-\gamma_1\gamma_2+\delta(\eta_1(\gamma_1-1)(\gamma_2-1)+\eta_2(\gamma_1-1)(\gamma_2-1)+2(\gamma_1+\gamma_2-\gamma_1\gamma_2))+\delta^2(\gamma_1+\gamma_2-\gamma_1\gamma_2-\eta_1(\eta_2-1)(\gamma_1-1)(\gamma_2-1)+\eta_2(\gamma_1-1)(\gamma_2-1))}{(1+\delta)^2} - c(\nu_1, \nu_2)$

Where $\gamma_1 = \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}$, $\gamma_2 = \frac{\alpha_2+\beta_2}{2\beta_2}$ and $c(\nu_1) = \frac{\nu_1^2}{2}$

$\nu_1 = ((-A + B)(-1 + d(-1 + h))(-1 + d(-1 + H)))/(4bB(1 + d)^2)$

$U_D = (1/(32b^2B^2(1 + d)^4))(-8ab(A - B)B(1 + d)^2(-1 + d(-1 + h))(-1 + d(-1 + H)) - 8b^2(A - B)B(1 + d)^2(-1 + d(-1 + h))(-1 + d(-1 + H)) + 16b^2B(A + B)(1 + d)^2(-1 + d(-1 + h))(-1 + d(-1 + H)) + (A - B)^2(-1 + d(-1 + h))^2(-1 + d(-1 + H))^2 - 32b^2B^2(1 + d)^2(dh(-1 + d(-1 + H)) - d(1 + d)H))$

$(0, \nu_2)$

$\nu_2 = ((-a + b)(-1 + d(-1 + h))(-1 + d(-1 + H)))/(4bB(1 + d)^2)$

$U_D = (1/(32(1 + d)^4))(16(1 + d)^2(-1 + d(-1 + h))(-1 + d(-1 + H)) + (8(a + b)(1 + d)^2(-1 + d(-1 + h))(-1 + d(-1 + H)))/b + (16A(1 + d)^2(-1 + d(-1 + h))(-1 + d(-1 + H)))/B -$

$(8A(a+b)(1+d)^2(-1+d(-1+h))(-1+d(-1+H)))/(bB)+((a-b)^2(-1+d(-1+h))^2(-1+d(-1+H))^2)/(b^2B^2)-32(1+d)^2(dh(-1+d(-1+H))-d(1+d)H))$

$(\nu_1, 0)$ preferred to $(0, \nu_2)$ iff $-(((a^2-A^2-2ab+b^2+2AB-B^2)(-1+d(-1+h))^2(-1+d(-1+H))^2)/(32b^2B^2(1+d)^4)) > 0$

This holds if $(-(\alpha_1+\alpha_2-\beta_1-\beta_2)(\alpha_1-\alpha_2-\beta_1+\beta_2)) > 0$

This is the same as the heterogeneous one of two from the baseline analysis

Version 2 Conditional on Elite Delivery

Second Position, first not coopted

$U_D(accept) = \eta_2 x_2 + (1-\eta_2)(\gamma_1+\gamma_2-\gamma_1\gamma_2)$

$U_D(reject) = \delta(\eta_2(1-y_2)+(1-\eta_2)(\gamma_1+\gamma_2-\gamma_1\gamma_2)) + (1-\delta)(\gamma_1+\gamma_2-\gamma_1\gamma_2)$

$U_E(accept) = \eta_2 y_2$

$U_E(reject) = \delta(\eta_2(1-x_2)$

$y_2 = \delta(1-x_2)$

$x_2 = \frac{\delta+\gamma_1+\gamma_2-\gamma_1\gamma_2}{1+\delta}$

Second Position, First coopted

$U_D(accept) = \eta_1\eta_2 x_1 x_2 + \eta_1(1-\eta_2)x_1 + \eta_2(1-\eta_1)x_2 + (1-\eta_1)(1-\eta_2)(\gamma_1+\gamma_2-\gamma_1\gamma_2)$

$U_D(reject) = \delta(\eta_1\eta_2 x_1(1-y_2)+\eta_1(1-\eta_2)x_1+\eta_2(1-\eta_1)(1-y_2)+(1-\eta_1)(1-\eta_2)(\gamma_1+\gamma_2-\gamma_1\gamma_2) + (1-\delta)(\eta_1 x_1 + (1-\eta_1)(\gamma_1+\gamma_2-\gamma_1\gamma_2))$

$U_E(accept) = \eta_1\eta_2 x_1 y_2 + (1-\eta_1)\eta_2 y_2$

$U_E(reject) = \delta(\eta_1\eta_2 x_1(1-x_2) + \eta_2(1-\eta_1)(1-x_2))$

$y_2 = \delta(1-x_2)$

$x_2 = \frac{\delta+\gamma_1+\gamma_2-\gamma_1\gamma_2+\eta_1(-\delta-\gamma_1-\gamma_2+\gamma_1\gamma_2+x_1+\delta x_1)}{(1+\delta)(1-\eta_1+\eta_1 x_1)}$

First Position

$U_D(accept) = \eta_1\eta_2 x_1 x_2 + \eta_1(1-\eta_2)x_1 + \eta_2(1-\eta_1)x_2 + (1-\eta_1)(1-\eta_2)(\gamma_1+\gamma_2-\gamma_1\gamma_2)$

$U_D(reject) = \delta(\eta_1\eta_2 x_2(1-y_1) + \eta_2(1-\eta_1)x_2 + \eta_1(1-\eta_2)(1-y_1) + (1-\eta_1)(1-\eta_2)(\gamma_1+\gamma_2-\gamma_1\gamma_2)) + (1-\delta)(\eta_2 x_2' + (1-\eta_2)(\gamma_1+\gamma_2-\gamma_1\gamma_2))$

$U_E(accept) = \eta_1\eta_2 y_1 x_2 + \eta_1(1-\eta_2)y_1$

$U_E(reject) = \delta(\eta_1\eta_2(1-x_1)x_2 + \eta_1(1-\eta_2)(1-x_1))$

$y_1 = \delta(1-x_1)$

$x_1 =$

$U_D = \eta_1\eta_2 x_1 x_2 + \eta_1(1-\eta_2)x_1 + \eta_2(1-\eta_1)x_2 + (1-\eta_1)(1-\eta_2)(\gamma_1+\gamma_2-\gamma_1\gamma_2)$

Investment (Coopt Both)

Objective Function: $1/4(-2v^2 - 2V^2 - ((2-h+d(4+2d(-1+h)(-1+H)-3H+h(-3+2H)))(a+b+v)(A-B+V))/(2bB(1+d)^2) + ((2-h+d(4+2d(-1+h)(-1+H)-3H+h(-3+2H)))(A+B+V))/(B(1+d)^2) + (1/(2(-1+d)(1+d)^2))(4-4h+4d(1+2h(-1+H)-3H)+4d^2(-1+h+H)+4d^3(-1-2h(-1+H)+2H) - \sqrt{(1/(b^2B^2))(-1+d)^2(16bB(1+d)^3(1-h)h(4bBd+2B(a+} 4vVX)$

FOCs:

$\frac{\partial}{\partial \nu_1} = 1/4(-4v - ((2-h+d(4+2d(-1+h)(-1+H)-3H+h(-3+2H)))(A-B+V))/(2bB(1+d)^2) - ((-1+d)(16bB(1+d)^3(1-h)h(-A+B-V) + 2(h+dh-dH)(A-B+V)((h+dh-dH)(a+v)(A-B+V) - b(B(1+d)(-4-4d+7h+8dh) + BdH + (h+dh-dH)(A+V))))))/(4b^2B^2(1+d)^2\sqrt{((1/(b^2B^2))(-1+d)^2(16bB(1+d)^3(1-h)h(4bBd+2B(a+b+v)+2b(A+B+V)-(a+b+v)(A} 4VX) = 0$

$\frac{\partial}{\partial \nu_2} = 1/4((2-h+d(4+2d(-1+h)(-1+H)-3H+h(-3+2H)))/(B(1+d)^2) - ((2-h+d(4+2d(-1+h)(-1+H)-3H+h(-3+2H)))(a+b+v))/(2bB(1+d)^2) - 4V - ((-1+d)(-16bB(1+d)^3(-1+h)h(-a+b-v) + 2(h+dh-dH)(a-b+v)((h+dh-dH)(a+v)(A-B+V) - b(B(1+d)(-4+7h+d(-4+8h)) + BdH + (h+dh-dH)(A+V))))))/(4b^2B^2(1+d)^2\sqrt{((1/(b^2B^2))(-1+d)^2(16bB(1+d)^3(1-h)h(4bBd+2B(a+b+v)+2b(A+B+V)-(a+b+v)(A} 4vX) = 0$

From figures, it appears that for low δ (and low γ_1, γ_2), coopting only the first elite is preferred to coopting both. In particular, when coopting the first elite, the dictator still has the outside option of coopting the second if bargaining fails. But if bargaining succeeds, he would not coopt the second? Why wouldn't the second elite just accept a lower offer? Check this.

I think what is happening here is that the amount the elite gets paid and whether are not they get paid are directly related. Even when both elites are coopted, the first elite will get more because of the bargain (unless $\eta_2 >> \eta_1$). x_1^* is increasing in η_1 and x_2^* is increasing in η_2 (equilibrium xs when both elites are coopted). The amount they get paid AND the likelihood that they get paid is increasing in η for each elite, so the dictator doesn't want to coopt both when η_2 is sufficiently high (and $\delta, \gamma_1, \gamma_2$) is low. So while the dictator would be better off coopting both even when he only needs on district to win because of the uncertainty, if the second elite is too expensive (because he is likely to deliver his district and the bargain (δ) and dictator's likelihood of winning without the elite advantages the elite) he will only coopt the first elite.

Coopt 1 only (but 2 is a backup if bargaining with 1 fails):

$U_D(accept) = \eta_1 x_1 + (1-\eta_1)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2)$

$U_D(reject) = \delta(\eta_1(1-y_1) + (1-\eta_1)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2) + (1-\delta)(\eta_2 x_2' + (1-\eta_2)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2)$

$y_1 = \delta(1-x_1)$ $x_1 = \frac{\delta^2\eta_1 + \delta\eta_2(\gamma_1-1)(\gamma_2-1) + \eta_1(\gamma_1+\gamma_2-\gamma_1\gamma_2) + \delta\eta_1(1+\gamma_1+\gamma_2-\gamma_1\gamma_2)}{(1+\delta)^2\eta_1}$

$U_D = \eta_1 x_1^* + (1-\eta_1)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2)$

Coopt 1 better than both iff

$(1/(2(-1+d)(1+d)^2))(-1+h-ho+d^3(1+2(-1+h)H(-1+o)(-1+p))+d(-1-(-1+2h)H(-1+o)(-1+p))-hp+hop+d^2(1+H(-1+o)(-1+p)+h(-1+o+p-op))+\sqrt{((-1+d)^2(-4(1+d)^3(-1+h)h(d+o+p-op)+(-1+d^2(-1+2h)+h(1+o+p-op)+d(-2+H(}$

0

Investment (Coopt 1 only)

Objective Function

$U_D = h*((dh + d^2h + dH + ho + dho - dHo + hp + dhp - dHp - hop - dhop + dHop)/((1 + d)^2h)) + (1-h)*(o + p - o*p)$ where $o = \gamma_1 = \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}$ and $p = \gamma_2 = \frac{\alpha_2+\beta_2}{2\beta)2}$ and $c(\nu_1) = \frac{\nu_1^2}{2}$ $= (1/(4bB(1+d)^2))(a(A-B)(-1+d^2(-1+h)+d(-2+h+H))-A(-1+d^2(-1+h)+d(-2+h+H))(b-v)+B((1-d^2(-1+h)-d(-2+h+H))v+b(3-2v^2+d(6+h+H-4v^2)+d^2(3+h-2v^2))))$

FOC:

$\frac{\partial}{\partial \nu_1} = (B - Bd^2(-1+h) - Bd(-2+h+H) + A(-1+d^2(-1+h)+d(-2+h+H)))/(4bB(1+d)^2) + ((-4bB - 8bBd - 4bBd^2)v)/(4bB(1+d)^2) = 0$

$\nu_1 = -((A - B + 2Ad - 2Bd + Ad^2 - Bd^2 - Adh + Bdh - Ad^2h + Bd^2h - AdH + BdH)/(4(bB + 2bBd + bBd^2)))$

$U_D = (1/(32b^2B^2(1+d)^4))((A-B)^2(-1+d(-2+d(-1+h)+h+H))^2 + 8bB(1+d)^2(b(1+d)((A+3B)(1+d)+(-A+B)dh)+b(-A+B)dH+a(A-B)(-1+d(-2+d(-1+h)+h+H))))$

(from figures, coopting both even without investment is better than coopting one with investment for some of the parameter space, so I really need to figure out the coopt both investment)

Version 3 Conditional on Dictator's Draw

2nd position, first not coopted

$U_D(accept) = \gamma_1 + \gamma_2 - \gamma_1\gamma_2 + (1-\gamma_1)(1-\gamma_2)\eta_2 x_2$

$U_D(reject) = \delta(\gamma_1 + \gamma_2 - \gamma_1\gamma_2 + (1-\gamma_1)(1-\gamma_2)\eta_2(1-y_2)) + (1-\delta)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2)$

$U_E(accept) = \eta_2 y_2$

$U_E(reject) = \delta(\eta_2(1-x_2)$

$y_2 = \delta(1-x_2)$

$x_2 = \frac{\delta}{1+\delta}$

2nd position, first coopted

$U_D(accept) = \gamma_1 + \gamma_2 - \gamma_1\gamma_2 + (1-\gamma_1)(1-\gamma_2)(\eta_2(1-\eta_1)x_2 + \eta_1(1-\eta_2)x_1 + \eta_1\eta_2 x_1 x_2)$

$U_D(reject) = \delta(\gamma_1 + \gamma_2 - \gamma_1\gamma_2 + (1-\gamma_1)(1-\gamma_2)(\eta_2(1-\eta_1)(1-y_2) + \eta_1(1-\eta_2)x_1 + \eta_1\eta_2 x_1(1-$
$y_2)) + (1-\delta)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2 + (1-\gamma_1)(1-\gamma_2)\eta_1 x_1)$

$U_E(accept) = \eta_2(1-\eta_1)y_2 + \eta_2\eta_1 y_2 x_1$

$U_E(reject) = \delta(\eta_2(1-\eta_1)(1-x_2) + \eta_2\eta_1 x_1(1-x_2)$

$y_2 = \delta(1-x_2)$

$x_2 = \frac{\delta+\delta\eta_1(x_1-1)+\eta_1 x_1}{(1+\delta)(1+\eta_1(x_1-1)}$

First Position

$U_D(accept) = \gamma_1 + \gamma_2 - \gamma_1\gamma_2 + (1-\gamma_1)(1-\gamma_2)(\eta_2(1-\eta_1)x_2 + \eta_1(1-\eta_2)x_1 + \eta_1\eta_2 x_1 x_2)$

$U_D(reject) = \delta(\gamma_1 + \gamma_2 - \gamma_1\gamma_2 + (1-\gamma_1)(1-\gamma_2)(\eta_2(1-\eta_1)x_2 + \eta_1(1-\eta_2)(1-y_1) + \eta_1\eta_2(1-$
$y_1)x_2) + (1-\delta)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2 + (1-\gamma_1)(1-\gamma_2)\eta_2 x_2')$

$U_E(accept) = \eta_1(1-\eta_2)y_1 + \eta_1\eta_2 y_1 x_2$

$U_E(reject) = \delta(\eta_1(1-\eta_2)(1-x_1) + \eta_1\eta_2 x_2(1-x_1))$

$y_1 = \delta(1-x_1)$

$x_1 =$

$U_D = \gamma_1 + \gamma_2 - \gamma_1\gamma_2 + (1-\gamma_1)(1-\gamma_2)(\eta_1\eta_2 x_1 x_2 + \eta_1(1-\eta_2)x_1 + \eta_2(1-\eta_1)x_2)$

$U_D = -1/(2(-1+d)(1+d)^2)((-1+d)(-((1+d)(-3+h+d(-3+2h))) + d(-3+2d(-1+h)+$
$2h)H)o(-1+p) + \sqrt{(-1+d)^2((1+d)^2(1+d-h)^2 + 2d(1+d)(-1+h+d(-1+2h))H + d^2H^2)(-1+o)^2(}$
$(-1+d)(-1+h+3p-hp+d(-2+3H+h(-3+2H)(-1+p)+6p-3Hp)+d^2(-1+2H+$
$2h(-1+H)(-1+p)+3p-2Hp)))$

Investment coopt both

Objective Function: $1/8(-4v^2 - 4V^2 - (1/(bB(-1+d)(1+d)^2))((-1+d)(-((1+d)(-3+h+$
$d(-3+2h))) + d(-3+2d(-1+h)+2h)H)(a+b+v)(A-B+V) - 2b(-1+d)(-((1+d)(-3+h+$
$d(-3+2h))) + d(-3+2d(-1+h)+2h)H)(A+B+V) + bB(4(-1+d)(-((1+d)(-1+h+d(-1+$

$2h)))+d(-3+2d(-1+h)+2h)H)+\sqrt{((-1+d)^2((1+d)^2(1+d-h)^2+2d(1+d)(-1+h+d(-1+2h))H}$

$8vVX)$

FOC: $\frac{\partial}{\partial \nu_1} = -v-((-((1+d)(-3+h+d(-3+2h)))+d(-3+2d(-1+h)+2h)H)(A-B+$

$V))/(8bB(1+d)^2)-\sqrt{((-1+d)^2((1+d)^2(1+d-h)^2+2d(1+d)(-1-d+h+2dh)H+d^2H^2)(a-b+v)}$

$d)(1+d)^2(a-b+v))-VX=0$

$\frac{\partial}{\partial \nu_2} = -V-(((-((1+d)(-3+h+d(-3+2h)))+d(-3+2d(-1+h)+2h)H)(a-b+v))/(bB)+$

$\sqrt{((-1+d)^2((1+d)^2(1+d-h)^2+2d(1+d)(-1-d+h+2dh)H+d^2H^2)(a-b+v)^2(A-B+V)^2)/(b}$

$d)(A-B+V))+8(1+d)^2vX)/(8(1+d)^2)=0$

Second Partials:

$\frac{\partial^2}{\partial \nu_1^2} = -1$

$\frac{\partial^2}{\partial \nu_2^2} = -1$

$\frac{\partial^2}{\partial \nu_2 \nu_1} = (((1+d)(-3+h+d(-3+2h))+d(3+2d-2(1+d)h)H)/(bB)-\sqrt{((-1+d)^2((1+d)^2(1+d-h)^2+2d(}$

$d)(a-b+v)(A-B+V)))/(8(1+d)^2)-X$

$\frac{\partial^2}{\partial \nu_1 \nu_2} = (((1+d)(-3+h+d(-3+2h))+d(3+2d-2(1+d)h)H)/(bB)-\sqrt{((-1+d)^2((1+d)^2(1+d-h)^2+2d(}$

$d)(a-b+v)(A-B+V)))/(8(1+d)^2)-X$

Hessian:

$$\left[(((1+d)(-3+h+d(-3+2h))+d(3+2d-2(1+d)h)H)/(bB)-\sqrt{((-1+d)^2((1+d)^2(1+d-h)^2+} \right.$$

Characteristic polynomial:

$(-1-\lambda)^2-((((1+d)(-3+h+d(-3+2h))+d(3+2d-2(1+d)h)H)/(bB)-\sqrt{((-1+d)^2((1+d)^2(1+d-h)^2+}$

$d)(a-b+v)(A-B+V)))/(8(1+d)^2)-X)^2=0$

both eigenvalues are negative with similar restrictions on χ and βs as above (χ must be sufficiently low, βs must be sufficiently high)

What about only coopting the first guy (and then only coopting the second if the first bargain fails)?

$U_D(accept) = \gamma_1 + \gamma_2 - \gamma_1\gamma_2 + (1-\gamma_1)(1-\gamma_2)(\eta_1 x_1)$

$U_D(reject) = \delta(\gamma_1 + \gamma_2 - \gamma_1\gamma_2 + (1-\gamma_1)(1-\gamma_2)(\eta_1(1-y_1)) + (1-\delta)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2 + (1-\gamma_1)(1-\gamma_2)(\eta_2 x_2')$

$U_E(accept) = \eta_1 y_1$

$U_E(reject) = \delta(\eta_1(1-x_1))$

$y_1 = \delta(1-x_1)$

recall $x_2' = \frac{\delta}{1+\delta}$

$x_1 = \frac{\delta(\eta_1+\delta\eta_1+\eta_2)}{\eta_1(1+\delta)^2\eta_1}$

$U_D = \gamma_1 + \gamma_2 - \gamma_1\gamma_2 + (1-\gamma_1)(1-\gamma_2)(\eta_1 x_1^*)$

Similar to V2: coopting only the first elite (with the second as the outside option) is preferred to coopting both when η_2 is sufficiently high (and $\delta, \gamma_1, \gamma_2$ are low) for the same reason as above

coopt one only preferred iff

$(1/(2(-1+d)(1+d)^2))((-1+h)(-1+o)(-1+p)-d^2(-1+h-H)(-1+o)(-1+p)+d^3(1+2(-1+h)H)(-1+o)(-1+p)-d(1+(-1+2h)H)(-1+o)(-1+p)+\sqrt{(-1+d)^2((1+d)^2(1+d-h)^2+2d(1+d)(-1$

0

Investment only one coopted

$U_D = (dh + d^2h + dH)/(1+d)^2 + ((A+B)(1-d(-2+d(-1+h)+h+H)))/(2B(1+d)^2) - v^2/2 + (((1-d(-2+d(-1+h)+h+H))/(1+d)^2 + ((A+B)(-1+d(-2+d(-1+h)+h+H)))/(2B(1+d)^2))(a+b+v))/(2b)$

$\nu_1 = ((A-B)(-1+d^2(-1+h)+d(-2+h+H)))/(4bB(1+d)^2)$

decreasing in α_2

decreasing in η_1

decreasing in η_2

decreasing in β_1

sign of $\frac{\partial \nu_1}{\partial \beta_2}$ depends on sign of α_2

$U_D(\nu_1) = (1/(32b^2B^2(1+d)^4))(32b^2B^2d(1+d)^2(h+dh+H) - 8ab(A-B)B(1+d)^2(1-d(-2+d(-1+h)+h+H)) - 8b^2(A-B)B(1+d)^2(1-d(-2+d(-1+h)+h+H)) + 16b^2B(A+B)(1+d)^2(1-d(-2+d(-1+h)+h+H)) + (A-B)^2(-1+d(-2+d(-1+h)+h+H))^2)$

$\tilde{\lambda}_{2,2} = \frac{\delta(\eta_2\gamma_1+\eta_1\gamma_2-\gamma_1\gamma_2(\eta_1+\eta_2))+\delta^2(\eta_1\eta_2(\gamma_1-1)(\gamma_2-1)+\eta_1\gamma_2+\eta_2\gamma_1-\gamma_1\gamma_2(\eta_1+\eta_2))}{(1+\delta)^2(\eta_1\eta_2(\gamma_1-1)(\gamma_2-1)+\eta_1\gamma_2+\eta_2\gamma_1-\gamma_1\gamma_2(\eta_1+\eta_2))}$