

Sowing Seeds of Destruction? Empowering Elite Rivals under Contested Dictatorship

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Abstract

Given the inherent risk, why would a dictator dole out policy control, riches, and military command to his rivals? Historical accounts focus on a dictator's underestimation of his rival's strength, failing to consider the perspective of the elite rival supporting him. In my theoretical model, a dictator faces a multi-stage process of conflict-eliminating elites from his ruling coalition to consolidate his power. The dictator can offer rewards in return for support, but empowering a rival limits his ability to purge them in the future. By introducing heterogeneity into the regime elite in a dynamic setting of uncertain conflict, I show the conditions under which both the dictator and his future opponent will be willing to work together *temporarily* despite the shadow of future conflict.

Keywords: Authoritarian Politics, Purges, Consolidation, Personalization, Coalitions

In the Soviet Union, when Joseph Stalin decided to move against his former allies in the Politburo, Lev Kamenev and Grigory Zinoviev, he rewarded his newest ally, Nikolai Bukharin, with a promotion as General Secretary of the Comintern Executive Committee in 1926. Bukharin's intellectualism and ideological rigor, as well as his popularity in the Party, helped legitimize the elimination of such powerful founding members of the Soviet system (Cohen, 1980). In Uganda, when Milton Obote found himself at odds with the entire Bugandan region because of his deposal of Mutesa of Buganda, he empowered Idi Amin, then deputy commander of the army, to take charge of military operations (Ingham, 1994, 103). Why would these leaders, threatened by a powerful and ambitious rival, empower that rival with positions of influence over policy or the military? Why would the elites enter into alliances with leaders trying to consolidate power by eliminating rivals?

These questions are particularly puzzling considering how disastrously these temporary alliances ended for the parties involved. After supporting Stalin in his earlier conflicts, Bukharin, too, was eventually expelled from the party in 1929, arrested in 1937, and executed in 1938. In Uganda, while President Obote was out of the country in January 1971, Amin led a coup, beginning a reign of terror that would last eight years. In each of these cases, both the leader and elite rival were jockeying for power, yet decided to form an alliance. These alliances went badly in the end as, ultimately, the goals of both members was always power.

When a dictator comes to power, the greatest threat to his tenure is members of his own regime (Meng, 2019; Myerson, 2008; Svobik, 2012). Eliminating the elites that make up his regime and consolidating more power for himself could not only protect the dictator from possible elite rebellion, but allow the dictator to implement his preferred policies, extract more resources from the state, and promote his own proteges. Dictators who successfully marginalize elites become leaders of "consolidated" or "personalist" style regimes (Gandhi and Sumner, 2019; Geddes et al., 2014; Svobik, 2009), yet this path to consolidation is not without risks. Successful consolidation does not occur without conflict: dictators purge elites

(Keller and Wang, Keller and Wang; Sudduth, 2017b) and elites counter the violation of the power-sharing status quo in a coup (Goldring, 2020; Luo and Rozenas, 2019; Meng, 2019; Sudduth, 2017a; Svobik, 2009). A dictator-elite conflict is far more likely to conclude in the dictator’s favor if he is powerful and his supporters numerous relative to the coupers attempting to depose him (Little, 2017; Singh, 2014). A more powerful elite opponent, in resisting the purge attempt, is more likely to defeat the dictator than a weak opponent. Thus if a power-hungry dictator wants to get rid of the members of his inner circle, why strengthen them with riches, positions, and prestige that could be used against him?

Sharing power¹ with the elites around him—whether through governmental or military positions, access to rents, or policy influence—has oft been thought to reduce conflict as the elites are less likely to coup (Paine, 2021; Svobik, 2012; Meng, 2019). While this maybe be true in some contexts, constrained peace, in which an agreement to share the spoils of rule among the dictator and elites is upheld, is not the only situation in which we should observe the dictator working together with members of his inner circle. As the above examples show, a dictator on his conflict-laden path to consolidation can ally temporarily with elites in order to improve his chances of removing others to consolidate his power. When the elimination of elite rivals is subject to uncertainty, rewarding a coalition member in order to entice him to the dictator’s side will help the dictator remove other elites in his way.

I model a multi-round contest in which a dictator can target a member of his regime for a purge then make an offer to the non-targeted elite to share the spoils. While working together makes his side more likely to win the current conflict, rewarding his elite ally enough to entice him into the uncertain conflict directly reduces the dictator’s chances of successfully eliminating that former ally in future consolidation attempts. Furthermore, the elite “ally” is willing to work with the power-hungry dictator, allowing the dictator to further grow the power that will be used against him in a future purge, for the right price. This model, in unpacking the process by which a dictator consolidates power, offers an alternative

¹I follow Arriola et al. (2017) and Meng (2019) view that access to material resources and prestige through governmental positions are the basis of power for the dictator and regime elites.

view of purges and elite eliminations (Keller and Wang, Keller and Wang; Sudduth, 2017b). While previous studies similarly recognize the risks that elite purges entail and the potential benefits to the dictator of eliminating surrounding elites, the potential for elite alliances with the dictator against their colleagues has yet to be explored. Such strategic elite action in which coalitions can occur with the dictator, instead of just among elites, are understudied in this burgeoning field and are necessary for explaining the elite rivals who succeeded in overthrowing leaders (Amin) and those who do not (Bukharin).

I show the conditions under which the dictator is willing to empower his most powerful elite in order to maximize his chances of successfully eliminating another member of the inner circle. When multiple rounds of conflict and consolidation are possible, the dictator will still prefer to empower a rival than fight alone—despite the direct impact it has on his likelihood of succeeding in the next round. The elite ally will accept a lower power-sharing offer from the dictator under the shadow of future conflict than a single round of coalitions. Due to the higher likelihood that he would win the conflict, the higher-powered elite will accept a lower power-sharing offer from the dictator than the lower-powered elite as the risk that such a payment must offset is lower. This framework of temporary coalition formation can be extended outside the world of authoritarian politics. In parliamentary democracies, we see brief coalitions among “strange bedfellows”—parties that share little ideological interest—despite their opposed electoral interests. Parties, firms, and individuals will work with their opponent not in spite of the anticipated conflict in their future, but because of it.

Coalitions and Consolidation

A dictator hoping to extend his tenure and govern with limited constraints seeks to establish a consolidated regime (Geddes et al., 2014, 2018). “Established” regimes are contrasted with those in which leaders are constrained by their elite allies that populate the upper echelons of the regime, often in military or party institutions (Svolik, 2009). Regimes and

leaders identified as unconstrained and “personalist” have lower failure rates than military regimes and military leaders (Geddes et al., 2014).² In addition to potentially lengthening his survival, an established leader benefits from the lack of constraint in other ways. More power vis-a-vis elites in the military or regime party allows a dictator to implement his preferred policies or promote his preferred personnel at will (Gandhi and Sumner, 2019; Meng, Meng). By eliminating elites within their own ruling coalition, a dictator can simultaneously increase his own power and policy control and deter challengers. Such personalized “despotic power”³ is often achieved by fully removing rivals from office, instead promoting personal loyalists to positions of power or even taking those positions for yourself. In Malaysia, for example, after removing his main rival Anwar Ibrahim from office, Mahathir Mohamad added to his already powerful portfolio of prime minister and home minister the posts of finance minister and, effectively, deputy prime minister (Slater, 2003). In Mali, Moussa Traoré added the ministry of defense and security and a position as the general secretary of the Democratic Union of the Malian People to his portfolio as president.

While removing or eliminating all rivals and potential threats in the elite population might be the best option in terms of power consolidation, budget and personnel constraints as well as the potential for a retaliatory coup may prevent the dictator from taking such extreme measures. If the dictator’s first priority is to retain office, as the loss of office could mean imprisonment, exile, or death, elite marginalization is a potentially risky endeavor (Svolik, 2012). Such blatant violation of the power-sharing agreement between the dictator and his allies will be met with resistance from the coalition (Svolik, 2009), creating regime instability that could end disastrously for the dictator (Luo and Rozenas, 2019). In addition to the general uncertainty that such conflict within the inner circle creates, there are also external sources of noise that could make the ex ante success of a purge attempt (or the responsive coup) unclear. Involvement in wars or international financial crises could suddenly shock

²While “consolidated,” “established,” and “personalist” are frequently used in different ways in the literature, the concepts all focus on a leader that is unconstrained by elites.

³Which Slater (2003) describes as “the power to decide” (82).

the regime’s access to financial and military resources. Domestically, intra-regime conflict outcomes further depend on people and whether they follow orders: uncertainty may stem from doubt over which side of the conflict the rank-and-file military or masses will support. Thus when choosing to initiate conflict with an elite member of the inner circle, the dictator, despite full knowledge of his own strength and the strength of the opposition, is uncertain about the outcome.⁴

How can a dictator bolster his chances of removing his rivals and achieve consolidation? Bringing elite allies into a coalition against a rival will help the dictator achieve the overwhelming advantage needed to eliminate an elite with minimal risk. As Obote needed support in countering the growing threat of Bugandans in his conflict with Mutesa of Buganda, promoting and empowering Amin, a potential rival, was the best way for him to maintain power initially (Ingham, 1994). Similarly, Stalin was not confident in his ability to remove the influential Kamenev and Zinoviev on his own: he took the time to build a coalition first to ensure their elimination spurred minimal backlash (Cohen, 1980). Here the dictator and his elite ally are working together and sharing power (even temporarily) not in order to avoid conflict (see Meng 2019), but in order to engage in conflict. By enticing one elite to support him against the other, the dictator is implementing a conventional “divide and rule” strategy (Acemoglu et al., 2004; Bates, 2014; Luo and Rozenas, 2019); however in this scenario it is not for the purposes of coup-proofing or preventing elites from becoming too threatening (Greitens, 2016), but in order to achieve his own ends of consolidation.⁵

When faced with a heterogeneous elite where members of the ruling coalition have their own networks, skills, power-bases, and ideologies, the question is not only when to entice an elite into joining the dictator against his colleague, but who? This is particularly important when elite elimination attempts are uncertain: while a more prominent and powerful elite

⁴This is not to say his expected chance of winning is 50-50. As described in the formalization below, the dictator’s likelihood of winning is increasing in his power advantage over his opponent, just with some stochastic noise.

⁵While allowing the elites to engage in an elite-initiated conflict against the dictator (coup) would let the dictator potentially make offers to elites in order to avoid conflict, this is outside the scope of the current model. The dictator entices an elite to his side in the conflict against a rival even without such complications.

could be more difficult to successfully remove from his position, a dictator who succeeds in doing so will receive greater benefits. I build on Acemoglu et al. (2008) model of non-democratic coalitions in which the relative power of coalition members determine who is removed by incorporating both that relative power and stochastic noise into an uncertain conflict. This uncertainty over conflict outcomes— and the risk-reward tradeoff that pursuing elite eliminations induces— is what motivates the dictator to willingly share power with a temporary ally to increase his likelihood of success despite the cost of compensating his coalition partner for the conflict risk they now share.

Having a coalition of powerful elites against a rival makes him more likely to succeed against his current opponent, but a dictator that is liberal with rewards is bargaining his future rivalries for his current conflict. Empowering an elite coalition member with greater access to military and financial resources, personnel, and policy influence will make him all the more difficult to purge later. However, if the dictator is unable to purge his opponent now, there will be no future conflict to be concerned about. His desire to consolidate now means the dictator may be willing to empower a rival, despite the effect on his future conflict success. Empowering a rival elite, especially under the shadow of future intra-regime conflict, may be a risk for dictators. On the path to personalization of power vis-a-vis elites, however, it is a risk that many dictators seem to be willing to take.

Elite Support for Dictatorial Consolidation

From the elite's perspective, why would an elite support a dictator on his path towards consolidation? Support does not come freely: elites who support the dictator against other elite threats are rewarded handsomely.⁶ If the dictator's offer is sufficient to make up for the risk of entering an uncertain conflict and possibly losing everything, a power-hungry elite will join such a coalition despite how much it will benefit the dictator. When the elite anticipates further conflict as the dictator continues to consolidate, we might expect him to

⁶Note that rewards here are a one-time promotion to a governmental position or financial gain, not a commitment to future loyalty and rewards upon which the dictator may have an incentive to renege.

avoid joining the dictator and easing his initial consolidation attempt. Why help your future attacker get more power? As I show below, by joining the dictator in his conflict against another elite, he not only bolsters himself with the power he gets from winning the first round conflict, he is reducing the dictator's advantage against him. If he will be attacked in the future anyway, an elite will be better off splitting the gains of conflict now with the dictator to increase his own power and reduce the dictator's (relative to if the dictator got to keep all the gains himself), supporting his own chances of survival.

Incorporating power heterogeneity into this coalition and conflict model is equally important from the elite's perspective: his identity not only determines whether he will be the dictator's target under certain conditions, but also how much he stands to gain from a successful elite elimination. Much like the dictator's consideration when determining whether to consolidate power through a purge, the elite joining the conflict takes into account the tradeoff between the reward from removing their colleague and the risk of being unsuccessful in the conflict. How much power their potential coalition partner, or potential enemy, brings to bear is a vital aspect of his decision to join the conflict and, if so, on which side.

I depart from existing explanations of elite-dictator coalitions in two important ways. First, by not tying the dictator's hands with a commitment to stop consolidating, power-sharing does not prevent future dictator-elite conflict (Meng, 2019; Myerson, 2008). Indeed, the threat of future conflict with the dictator will make the elite **more** likely to accept power from the dictator as such a transfer will make him more advantaged in the future conflict. Second, while conflict outcomes are uncertain (though a function of the relative power of the involved individuals), the dictator and elites all have full information about each others' power and intentions. Many historical explanations of elite-dictator coalitions and conflict between the same individuals suggest that they had incomplete information about each others' ambitions or their relative power and ability to eliminate the other (Ingham, 1994; Kershaw, 2014). In this line of reasoning, for a dictator to willingly strengthen his rival he must underestimate the rival's strength. But for the rival to be willing to support the

dictator in his path of consolidation and help him achieve even more power, the rival must overestimate his own strength. Even in low-information environments, such divergent prior beliefs would be insupportable.⁷

While the opacity of authoritarian politics can lead to misperception over rivals' strengths, incomplete information is not the only explanation for conflict in the inner circle. I argue that a dictator, fully informed of the power and threat that his rival presents, is still willing to empower the elite in order to ensure his victory in the early stages of consolidation against other elites. Further, my theory simultaneously explains an elite's willingness to join a coalition with a dictator with whom he will be in conflict in the future. I show that under certain distributions of power among the ruling coalition, both the dictator and his future opponent are willing to work together *temporarily*, for the right price, despite the shadow of future conflict. It is not the case that Stalin was able to manipulate and "trick" Bukharin in a way that Obote was unable to do with Amin: both dictators chose to empower elites to support them against other elite problems (Kamenev and Zinoviev or Mutesa of Buganda, respectively) despite the uncertainty it created for their future consolidation.

Model Setup

Three players, a dictator (D), an elite with a high initial endowment of power (H), and an elite with a lower initial endowment of power (L), together form a regime. Each player's type, τ_i , is his power endowment. H's initial endowment of power is fixed at H ($\tau_H \equiv H$); the dictator's endowment is $d > 0$ greater than H's ($\tau_D \equiv H + d$) and L's endowment is $w \in (0, H)$ less than H's ($\tau_L \equiv H - w$). The dictator has the most power initially (which is why he is the dictator) and the high elite has more initial power than the low elite. Thus d

⁷This is not to say that incomplete information regarding individuals' strength and conflict actions is impossible. Indeed, in some cases a lack of information may lead to conflict. My model shows that incomplete information is not the only explanation for conflict and may be more applicable in regimes where there are institutions with the purpose of avoiding such information-induced conflicts (Boix and Svobik, 2013) and yet conflicts are still observed (see Luo and Rozenas (2019) for a summary of elite-dictator conflicts despite institutions).

can be interpreted as the dictator’s initial power advantage over the other two elites while w denotes the relative difference between elites H and L.

Play takes place over the course of two rounds. First, the dictator chooses which elite to target for elimination or does not initiate conflict, ending the round. If a target is chosen, the dictator can make a take-it-or-leave-it offer, $x \in [0, \bar{x}]$, his post-conflict budget constraint, to the non-targeted elite to share the spoils of the conflict (conditional on winning) with that elite in exchange for the elite’s support in a coalition against the target.⁸ The dictator’s budget, \bar{x} , is the total amount of power he would have if he won the conflict: the combined power of the dictator and his target. The non-targeted elite can choose to accept the dictator’s offer and join a coalition with the dictator, join the targeted elite, or remain out of the conflict. The dictator (or coalition) and target(s) then participate in a contest where the probability that each side wins is the difference between their relative power plus mean-zero noise; e.g. participant (or coalition) i wins the conflict if $\tau_i \geq \tau_j + \epsilon_t$ where $\epsilon_t \sim U[-a, a]$ independent of the round. Thus the probability that participant or coalition i wins is $F_\epsilon(\tau_i - \tau_j)$. While the likelihood of winning is based on the power differential between the opponents, I assume all conflicts are uncertain.⁹ If a coalition formed, whichever side won the conflict splits the power of the loser(s) according to the agreed upon division. If no coalition formed, all of the loser’s power is transferred to the winner.

The first round ends, and whichever remaining player has the most power now becomes the dictator and can choose to initiate conflict with the other remaining member of the regime (if there is a second member—if there is only one player remaining in round two, the game ends). If three players remain, the dictator may make a coalition offer as above. If conflict is initiated, it occurs as previously described, with a new, independent draw of ϵ . The power of the loser(s) is transferred to the winner(s), potentially shared among a coalition if they win together.

⁸I analyze an alternative bargaining protocol in the appendix for robustness.

⁹In particular, a is sufficiently large.

Sequence of Play

$t = 1$

- The dictator, D chooses to target H, target L, or no target. If no target is selected, the round ends.
- The dictator, D, chooses an offer $x \in [0, \bar{x}_1]$ to make to the non-targeted elite. \bar{x}_1 is the dictator's post-conflict budget constraint in the first round, which depends on the selected first round target.
- The non-targeted elite chooses whether to participate in the conflict on the side of the dictator, the target, or not participate.
- The dictator (or coalition) and target(s) participate in a contest where participant(s) i win if $\tau_i \geq \tau_j + \epsilon_1$ where $\epsilon_t \sim U[-a, a]$
- Power is transferred from the loser(s) to the winner(s), split among a winning coalition if an offer greater than 0 was made and accepted. Whichever remaining player now has the most power is the dictator. If only one player remains, the game ends.

$t = 2$

- The dictator chooses to target one of the remaining players, or no target. If no target is selected, the game ends.
- If three players remain, the dictator chooses an offer $x \in [0, \bar{x}_2]$ to make to the non-targeted elite.
- If there is a non-targeted elite, he chooses whether to participate in the conflict on the side of the dictator, the target, or not participate.
- The dictator (or coalition) and target(s) participate in a contest where participant(s) i win if $\tau_i \geq \tau_j + \epsilon_2$ where $\epsilon_t \sim U[-a, a]$

- If the dictator (or dictator’s coalition) wins, the loser(s)’s power is transferred to the winner(s), split according to the accepted offer if a coalition formed. The game ends.

Payoffs

All members of the ruling group derive utility from their endowments of power at the end of the game, which are a function of the dictator’s target choice(s), offer(s), and the coalition decisions of the elites across both rounds. These actions are, in turn, a function of each individual’s relative power and the uncertainty of the conflict environment.

$$u_i(H, d, w, a) = \tau_{i,t=2}$$

This formalization of elite-dictator conflict and coalition formation yield a few substantive scope conditions. First, the dictator, as the conflict “agenda-setter,” must be able to target an elite for a purge. This could include introducing articles of expulsion to a legislature, ordering police to make an arrest, or prompting a paramilitary to attempt assassination. I do not assume is that the dictator must be such a powerful strongman that he can unilaterally remove and elite from the position in the inner circle and usurp their power with certainty. Second, the dictator commits to his course of conflict before making an offer to the non-targeted elite. If the elite rejects his spoil-sharing coalition offer, the dictator will still engage in conflict with the purge target. This is similar to a public accusation of treason or corruption or a movement to arrest the target before any attempts at coalition formation.¹⁰ Third, the dictator may empower his rival by sharing the spoils of conflict, but the elites do not have the option to make lasting commitments to one another about the division of power if they overthrow the dictator together. A full change in leadership, as would occur if the elites successfully overthrow the dictator, is a sufficiently destabilizing that a newly installed leader is unlikely to follow any pre-coup commitments to power sharing (Sudduth, 2017b). While elites can form a coalition against the dictator, the model precludes power-sharing

¹⁰The alternative approach is discussed in the bargaining version in the appendix.

commitments among the elites to isolate dictator's incentives to empower a rival. Lastly, in order to focus on the dictator's conflict-initiation and power-sharing with elite rivals, elite-initiated conflicts (i.e. coups) are not possible. Similar to other models of authoritarian power sharing, it is the dictator's initiative to violate the power-sharing agreement that leads to the conflict, not a pre-emptive elite coup (Meng, 2019; Svulik, 2009).

Results

The game is solved using backwards induction for sub-game perfect Nash equilibria. Note a second round in which all three players remain is equivalent to a one-shot version of the elimination game in which there is no shadow of future conflict.

Three Remaining Players

The last action of the game, and therefore the first to be addressed, is the non-targeted elite's decision to join a coalition or stay out of the conflict. Due to the uncertain nature of all conflicts, if no offer is made to him, either elite would prefer to stay out of the conflict and maintain his status quo power rather than risk a conflict for no additional benefit. An offer of power that the dictator makes to the non-targeted elite must be high enough to compensate the elite for the risk he is taking on by joining the conflict; I term this minimum power needed to induce an elite to join the conflict the *join condition*, stated in Result 1. As the elites are heterogeneous, they each have their own thresholds that are a function of both their status quo power (what they would keep if they stayed out of the conflict), and the probability that their coalition with the dictator would successfully defeat the targeted elite. The dictator, however, is not necessarily willing to share sufficient power to get an elite to join him. Instead, the dictator has a maximal offer, termed the *offer condition*, he is willing to make to the non-targeted elite; if he had to pay any more than this maximum, he would rather just fight alone (see Result 2).

Result 1. *If L is targeted, H will join the dictator if D offers $x_H \geq \underline{x}_H \equiv \frac{H(1-F_\epsilon(H+d+w))}{F_\epsilon(H+d+w)}$, otherwise he will not join the conflict. If H is targeted, L will join the dictator if D offers $x_L \geq \underline{x}_L \equiv \frac{(H-w)(1-F_\epsilon(H+d-w))}{F_\epsilon(H+d-w)}$ otherwise he will not join the conflict.*

Result 2. *If L is targeted, the maximum offer D is willing to offer H is $\bar{x}_H \equiv \frac{(2H+d-w)(F_\epsilon(H+d+w)-F_\epsilon(d+w))}{F_\epsilon(H+d+w)}$. If H is targeted, the maximum offer D is willing to offer L is $\bar{x}_L \equiv \frac{(2H+d)(F_\epsilon(H+d-w)-F_\epsilon(d))}{F_\epsilon(H+d-w)}$.*

Given the expected coalition decisions and minimal offers needed to induce such coalitions, will the dictator initiate conflict? If so, which elite will he target? High conflict uncertainty benefits the target and mitigates the dictator's power advantage. Even when a coalition member would increase his chance of victory, an uncertain conflict environment means that the increase in win probability is not large enough to counteract the decrease in benefits from the offered power transfer. The dictator prefers maintaining his power and avoiding risky conflict, therefore he chooses no target. If the dictator's advantage is great enough relative to the conflict uncertainty, however, the dictator is willing to choose a target and make a positive offer of power sharing to the non-targeted elite. Under this level of uncertainty, both coalitions are available: the dictator could come to a power-sharing agreement with either H or L. Additionally, both of these target and coalition options are strictly preferred to no conflict. In choosing between targets and the resulting coalition, the dictator chooses to target L and make a sufficient coalition offer to H.

Proposition 1. *If the dictator's advantage relative to the elites and uncertainty is sufficiently high ($d > a + w - 3H$), D will target L and make H a sufficient offer \underline{x}_H such that H joins the dictator's coalition. If the dictator's relative advantage is sufficiently low ($d < a + w - 3H$), the dictator will choose no target and no conflict will occur.*

Because H is more powerful, the dictator is maximizing his win probability by joining forces with H ($F_\epsilon(H+d+w) > F_\epsilon(H+d-w)$). Despite a lower benefit from conflict in terms of elite elimination (defeating L gives the dictator less power than defeating H), H's minimal sufficient power-sharing offer is lower than L's (because of his higher likelihood of winning)

so the dictator does not need to transfer as much. Note that the dictator is empowering his most serious rival, the most powerful elite in the regime, in order to support his attack on the least powerful member of the regime. At this point, however, there is no shadow of future conflict that the dictator must guard against: after they defeat L, D and H share power without issue. This changes below, when future conflict alters the incentives of both the dictator and his coalition partner.

This general result—that the dictator will coalesce with the lower-powered elite against the higher-powered elite for a large portion of the parameter space—is robust to an alternative bargaining protocol in which the dictator does not commit to a purge target. In this alternating offer Rubinstein bargaining version, the dictator can bargain with both elites and thus use the threat of coalescing with the *other* member of the regime to fight the elite in the negotiation. This alternative protocol and how the results mirror Proposition 1 are discussed in the Appendix.

Two Remaining Players

If there are two players remaining in the second round, there must have been a first round conflict that eliminated one of the members of the regime.¹¹ Regardless of who the second round dictator is, regardless of how much power was transferred after the first round, regardless of the identity of the remaining regime member, the dictator will target the remaining regime member for conflict if uncertainty is sufficiently low relative to the dictator's advantage. Note that the dictator's advantage is no longer necessarily D's advantage: the power d now belongs to whoever won the first round conflict and is now the second round dictator. If the conflict environment is too uncertain, the dictator (whoever he is) will maintain the power he gained from the first round rather than initiate a second round conflict.

¹¹There are four ways for there to be exactly two players remaining: (1) H and D fought alone (or L joined and received no transfer); (2) L and D fought alone (or H joined and received no transfer); (3) a coalition of D and L fought H, so now D and L remain with some power transferred to L; (4) a coalition of D and H fought L, leaving D and H with some power transferred to H. If a sufficient offer was made and a coalition formed in the first round and won, some power was transferred to the elite member of the coalition according to the bargain.

Proposition 2. *If the dictator's advantage relative to the elites and uncertainty is sufficiently high ($d > a + w - 3H$), the dictator will target the remaining regime member. If the dictator's relative advantage is sufficiently low ($d < a + w - 3H$), the dictator will choose no target and no conflict will occur.*

First Round Offers and Targets

The first round of conflict must take into account the potential for conflict in the second round: there is a shadow of future conflict affecting the players' decisions to join coalitions, make offers, and choose targets. Sharing power with an elite is now sharing power with a future rival, if conflict will occur in the second round. Based on the above result, we know that if uncertainty is particularly high, no conflict will occur in the second round regardless of which players remain. With no anticipated second round conflict, the elites' decision to join the dictator is the same as the one-shot version: if the dictator was to make him a sufficient offer, the non-targeted elite would join his coalition. However, in this parameter range, the dictator is not willing to make such an offer; furthermore, the dictator is not even willing to initiate a one-on-one conflict with either elite.

Proposition 3. *If the dictator's advantage relative to uncertainty is sufficiently low ($d < a + w - 3H$), there will be no conflict. The first round join conditions for the elites are the same as three players in round two ($\underline{x}_H, \underline{x}_L$), but the dictator will not initiate any conflict.*

If the dictator's advantage is sufficiently greater than the conflict uncertainty, there will be conflict in the second round, the anticipation of which must be taken into account in both the targeting and offer decisions of the dictator as well as the coalition decision of the elite. Recall that any offers and power transfers made in the first round directly affect the conflict success probabilities in the second round. By joining the dictator's coalition in the first round, the non-targeted elite is taking on the risk of not only the first round conflict, but the second round conflict in which he, if his coalition won in the first round, will definitely be a participant. However, the potential benefits of such conflict involvement are extreme:

instead of keeping his own power plus whatever transfer the dictator offered him as was the case in the one-shot version, the elite now has the opportunity to win all the combined power of the regime.

Result 3. *If the dictator's relative advantage is sufficiently high ($d > a + w - 3H$): if L is targeted, H will join the dictator if D offers $x_H \geq \underline{x}'_H \equiv \frac{(a-H-d-w)(a-H-d+w)}{2(H+d+w+a)}$, otherwise he will not join the conflict. If H is targeted, L will join the dictator if D offers $x_L \geq \underline{x}'_L \equiv \frac{(a-H-d-w)(a-H-d+w)}{2(a+H+d-w)}$, otherwise he will not join the conflict. In this parameter range, $\underline{x}_H > \underline{x}'_H$ and $\underline{x}_L > \underline{x}'_L$.*

As before, the elites are not willing to join the dictator and take on the risk of a first round conflict without some compensation. If they do not join him, they will still be targeted for conflict in the second round and the full regime's power is still winnable. Why, then, would a non-targeted elite join a first round conflict on the side of the dictator knowing he will have to fight a second round conflict as well? While the ultimate prize is the same regardless of whether he fights both rounds or stays out until being targeted in the second round, the probability that he wins is not the same. By accepting a power transfer and taking on some of the risk of the first conflict, the non-targeted elite is not only increasing his power in the second round conflict but **decreasing the dictator's**. If he stays on the sidelines, whoever wins the first round will have a large power advantage over him because they would have the combined power of both the dictator and the first target. Taking a power transfer doubly reduces this power differential as it both increases the non-target's power by x_i and decreases the dictator's power by x_i . The elite's chance of winning the second round increases enough to not only make joining the dictator worth the risk, but to make the minimal sufficient offer lower than his join condition in a one-time conflict.

The dictator's decision is two-fold: he must choose who to target (if anyone) as well as whether to make the non-targeted elite their minimal sufficient offer. While the dictator does reduce his chances of winning the second round conflict by making a transfer to the non-targeted elite (who is his future opponent), no second round conflict would occur (for

him) if he does not win the first. The dictator is willing to make a minimal sufficient offer to the non-targeted elite in order to increase his chances of winning the first round conflict despite the mitigation of his second round advantage. Note from above that $\underline{x}'_H < \underline{x}'_L$: the minimal sufficient offer to get H to join the dictator's coalition is less than the minimal sufficient offer to get L to join his coalition. As H is already closer to the dictator in power than L, he does not need as high of a transfer to agree to join the conflict. Thus while D's initial power advantage over L is greater, the higher transfer that must be made to L reduces this advantage to the point that the dictator is perfectly indifferent between initially targeting H and making L an offer and initially targeting L and making H an offer. While a coalition between H and D is stronger in the first round, D's probability of winning the second round is lower, so the total expected utility is the same as an initial coalition between L and D.

Proposition 4. *Assume the dictator's relative advantage is sufficiently large ($d > a + w - 3H$). Offering a minimal sufficient transfer to induce a coalition $(\underline{x}'_H, \underline{x}'_L)$ dominates targeting an elite and failing to form a coalition. When uncertainty is sufficiently low, targeting an elite and making a minimal sufficient offer dominates no conflict initiation.*

While making an offer and a coalition is preferred to fighting alone, it is not necessarily preferred to initiating no conflict whatsoever. As uncertainty approaches the upper case bound, no target is preferred. On the lower end of uncertainty, however, the dictator prefers to initiate conflict with one of the elites, forming a coalition with the other, rather than no conflict. Recall, however, that he is indifferent between his target and coalition options. Joining forces with the most powerful elite regime member is no longer a dominant strategy as it was in the one-shot version.

Discussion

Why would a dictator and elite work together when trying to consolidate more power for themselves at the expense of one another? When greater advantage over an opponent increases the likelihood of succeeding in conflict, the dictator has an incentive to build a coalition against his purge target despite the compensation that coalition members require. In one round of conflict, the dictator empowers his most powerful elite rival in order to maximize his likelihood of purging successfully and resisting the target's counter action at a lower cost. More importantly, both the dictator and the elite will utilize this temporary alliance to defray the risks of the current conflict when conflict is expected in the future. The dictator will empower his rival with full knowledge that they will fight in the future. The elite will support the dictator in the first round knowing he is next to be purged in order to better his chances of winning. This intuition offers an alternative explanation for the behavior of elites facing a dictator consolidating power: it wasn't that Bukharin naively expected to be safe from Stalin as he watched his colleagues succumb to the Mass Terror, he was siding with Stalin to garner enough resources and power to make his own eventual downfall a fairer fight.

This framework of elite-dictator conflict supports a variety of possible extensions to deepen our understanding of the inner circle. For simplicity, the loser's power is transferred to the winner(s) in its entirety. While dictators do often take the official positions in the party and government for themselves (e.g. Saddam Hussein, Moussa Traoré) or purposely maintain vacancies (e.g. Félix Houphouët-Boigny), they cannot truly rule alone. The replacement of purged elites could take multiple forms. Most simply, the successful side of the conflict could take some portion of the power that the eliminated party held and a replacement of lesser power is promoted into the inner circle. This would reduce the parameter space over which the dictator will initiate conflict (because his expected benefit of conflict is lower) and reduce the amount of power he is willing to share with his coalition member. In a dynamic conflict environment, replacement of regime members would beget

evermore conflict: as the dictator's power grows with each purge and his opponents weaken, the level of stochastic conflict risk needed to deter him from conflict would grow. What if the replacements were not necessarily weak? Like other models of personnel (Egorov and Sonin, 2011; Montagnes and Wolton, 2019), the replacement could be of an unknown type (in this case, an unknown level of power). If the dictator attempts to purge his current rival, the replacement member of the inner circle could be more powerful. This is not necessarily a conflict deterrent, however. A more powerful dictator will initiate conflict with a more powerful opponent when exogenous uncertainty is sufficiently low. Indeed the possibility of being able to take the power of a future opponent with greater rewards could make the dictator more likely to initiate conflict as such a windfall would only be available if he removed his initial allies.

Similar logic applies to exogenous shocks to the inner circle's power. If there was uncertainty about the future relative power of the regime-members, conflict will not universally increase (or decrease). While an extremely weak dictator (relative to uncertainty) may still prefer to maintain his status quo power until his rival experiences a negative shock, a dictator of middling strength would anticipate a positive power shock to his opponent by initiating conflict now in order to be at full strength against a more powerful rival and capture those greater rewards. From the elites' perspective, being compensated in a winning coalition is more vital to their future success if a shock to their power is anticipated. The results would be impacted most if the heterogeneity of the elites extended to their likelihood of receiving a power shock as this would impact which elite the dictator would target first and which he would rather ally with first.

Conclusions

Why would a dictator, limited by the elites around him, empower a rival with whom he will fight in the future? Why would an elite form a coalition with a dictator attempting to

consolidate power, easing his path to consolidation? While historians have pointed to the underestimation of the rival's strength or overestimation of the rival's loyalty as possible explanations, my theoretical model explains both the dictator's empowerment of a rival and the rival's willingness to support a dictator on a path of consolidation. What does drive the willingness of a dictator and his rival to work together is the general uncertainty that the dictator faces in his path to consolidation. Sudden changes in finances, mass sentiment, or arbitrary difficulties preventing arrest and detention can make an attempt at elite elimination fail. The dictator, however, can mitigate these uncertainties when he builds a coalition of overwhelming power. Thus the dictator is willing to take a risk and empower a rival in order to increase his chances of consolidation now, knowing full well that his future conflict with said rival will be affected. The elite rival, aware that he will have to face the dictator in a conflict regardless, would rather take the additional power being offered, making the future conflict more even between the two parties.

The path to dictatorial consolidation of power is littered with risks, but the potential rewards of being an unchallenged ruler, in control of policy, personnel, and resources, may be worth the conflict. For dictators with multiple elite rivals to remove, forming transitory coalitions with rivals can be the best strategy for risk mitigation. As was the case with Stalin's alliance with Bukharin, working with an elite rival was needed to successfully eliminate other rivals; in the end, such dictators are still able to achieve high levels of personalization, removing those rivals who they had previously empowered. Elite empowerment is not without risk, however, as Obote found with his disastrous reliance on Idi Amin.

By formalizing a dynamic consolidation process with global conflict uncertainty, I have furthered our understanding of elite-dictator relations with both coalitions and conflict. Institutions that reduce informational asymmetries among the dictator and elites does not necessarily imply stable, long-lasting power-sharing agreements. Even with complete information about the relative strengths of potential opponents, conflicts and coalitions of convenience can still emerge and a dictator may still successfully personalize power. The

underlying balance of power and global conflict uncertainty, not only asymmetrical information about the strength of each actor, can explain not only the seemingly bizarre behavior of elite coalition behavior against their own colleagues during periods of intra-regime conflict, but the dictator's willingness to dole out offices, money, *dachas*, and other goods to his supporters only to seize them upon the next round of treason accusations.

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Appendix A: Alternative Bargaining Protocol

Is it reasonable that the dictator can make a take it or leave it offer to an elite? Is it reasonable to assume that the dictator is committed to a conflict target regardless of whether he induces the support of the other elite? While the mechanic of divide and rule (targeting one, coalition with the other) occurs in the above model, allowing the dictator to play the elites off of one another with the threat of being the purge target in the same round captures the spirit of divide and rule.

Model Setup

Three players, a dictator (D), an elite with a high initial endowment of power (H), and an elite with a lower initial endowment of power (L), together form a regime. Each player's type, τ_i , is his power endowment. H's initial endowment of power is fixed at H ($\tau_H \equiv H$); the dictator's endowment is $d > 0$ greater than H's ($\tau_D \equiv H + d$) and L's endowment is $w \in (0, H)$ less than H's ($\tau_L \equiv H - w$). The dictator has the most power initially (which is why he is the dictator) and the high elite has more initial power than the low elite. Thus d can be interpreted as the dictator's initial power advantage over the other two elites while w denotes the relative difference between elites H and L.

The dictator chooses which elite to bargain with first. He makes the first elite an offer $x_i \in [0, 1]$ to share the spoils of conflict, the power of elite $-i$, where the elite i keeps $x_i(\tau_{-i})$ and the dictator keeps $(1 - x_i)(\tau_{-i})$. If elite i accepts, they together fight elite $-i$ in a contest where the probability that each side wins is the difference between their relative power plus mean-zero noise; i.e. coalition dictator and i wins the conflict if $\tau_D + \tau_i \geq \tau_{-i} + \epsilon_t$ where $\epsilon_t \sim U[-a, a]$ independent of the round. If no agreement is reached (bargaining breaks down), the dictator will then bargain with the other elite, $-i$, making an offer of $x_{-i} \in [0, 1]$ of sharing the spoils of conflict against elite i . If they agree, the contest occurs as described above with the dictator and his coalition partner $-i$ against elite i . If this second bargain

does not reach an agreement, the dictator has the choice to fight either elite on his own (using the same contest function with noise) or not initiate conflict.

I utilize an alternating offer Rubinstein bargaining protocol with risk of breakdown. Whenever an offer of division is rejected, with probability $\delta \in [0,1]$ the other party makes a counteroffer or bargaining breaks down and the dictator must stand for a mass election without elite support in the district with probability $1 - \delta$. If the dictator and elite fail to reach a successful bargain (or bargaining breaks down before they are able to come to an agreement), the dictator moves to the next elite. When no elites remain, the dictator chooses whether to initiate a one-on-one conflict, keeping all the spoils of the elite elimination for himself if he succeeds, or chooses no target and does not initiate conflict. Nature then draws ϵ , the conflict occurs (if there is one), and payoffs are distributed.

Results

The relevant equilibrium is subgame perfect Nash. The dictator and each elite utilize stationary strategies. The dictator proposes x proportion of the conflict spoils to the elite, keeping $1 - x$ for himself, every period and accepts the elite's proposal if and only if $y \geq y'$. The elite proposes y proportion of the conflict spoils to the dictator, keeping $1 - y$ for himself, every period and accepts the dictator's proposal if and only if $x \geq x'$. The outside options (what the expected utilities of the players are if bargaining fails) are determined by what the dictator will do at the end of the game if no coalitions form as well as the other elite's expected behavior. In equilibrium, an agreement is reached between the first elite and the dictator who then fight the other elite. By choosing the bargaining order, the dictator is choosing who to coalesce with and who to target.¹²

The full equilibrium characterization is in the formal appendix (it is defined by the optimal offer and counter offer for the dictator and elite in each individual bargain, bargaining order subgames, and the dictator's outside option (lone targeting decision) and is therefore

¹²But more indirectly than in the previous model. Indeed it is the off-path option of targeting/coalescing with the other player that affects the split of spoils.

very long formally). The dictator has a different targeting/coalition strategy depending on his advantage relative to the elites and conflict uncertainty that defines his true outside option (unilateral conflict). When the dictator is relatively weak ($a \in (H + d + w, 2H - w)$), if he was going it alone he would target L, the weaker elite. In this case, he prefers to bargain with L first, using the threat of joining with H against him, inducing him into a coalition against H. A dictator of middling strength relative to uncertainty ($a \in (2H - w, 2H + d)$) would target H for a one-on-one conflict. He prefers to bargain with H first, using the threat of fighting him with L, inducing him into a coalition against L. A dictator in a highly uncertain conflict environment ($a > 2H + d$) would avoid conflict if he had to go it alone. When he can play the elites off one another, he bargains with L first and uses this coalition against H. How the dictator's equilibrium strategy changes as uncertainty increases can be seen in Figure 1.

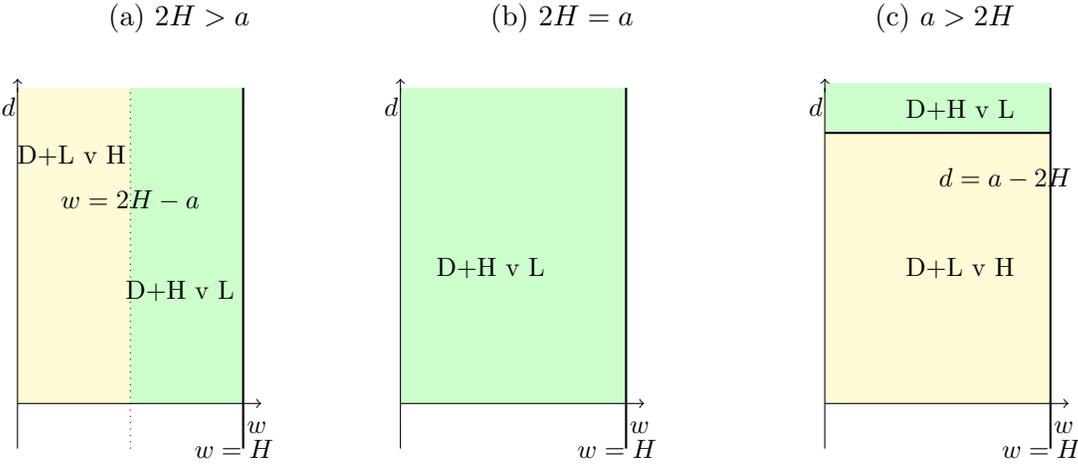


Figure 1: Single Conflict with Bargaining for Coalition

Comparing Bargaining Protocols

These two approaches (take it or leave it offer with conflict commitment versus bargaining) have different potential drawbacks and benefits. In the bargaining version, the dictator is able to play the elites off of one another because he has the option of bargaining with the other person— he has not committed to a target yet. For most of the parameter space (outside

of the tails of extremely low uncertainty and high uncertainty), the dictator's coalition and targeting behavior is the same as the main model: he makes an offer to, and forms a coalition with, H and they fight L.

This bargaining protocol, in which the dictator can credibly threaten to not only target the elite to whom he is making an offer but fight the elite with a coalition including the third member of the regime, clearly benefits the dictator. Because the likelihood of loss is higher if the dictator's offer is refused is significantly higher than it is with the one-shot take it or leave it offer discussed in the main text, the dictator can induce a coalition across the full domain of uncertainty. Thus in the parameter space where, in the main model, we would expect the dictator to not initiate any conflict in order to protect his status quo power, instead the dictator will create a coalition with the low-powered elite, L, against H.

While this protocol is more generous to the dictator in his ability to play the elites off of one another as potential targets, limiting the options of the elites to only accepting or rejecting the dictator's offer may not be fair. Not only does this protocol preclude elites from joining one another in the coalition, it also does not allow them to stay outside the conflict as, on path, they will either be an aggressor or a target. With this dichotomous choice—aggressor or target—it is not surprising that each elite would accept the dictator's offer to join his coalition for some compensation. When the dictator is initiating a purge, why would an elite join him in a coalition? This bargaining protocol shows that the elite will join to avoid being a target in addition to whatever spoils the dictator shares. In the main model, however, the elite is willing to join the dictator without the additional inducement of being a potential target of the purge. Instead, it is only the offer to share the victim's power that induces the elite to join a coalition. It is a harder case to induce the elite into a coalition. As the objective is to consider the perspectives of the dictator empowering a rival and the elite supporting a power-hungry dictator, the bargaining protocol that is more conservative in inducing the elite coalition best supports this objective.

Appendix B: Main Model

Round 2: Three Players Remaining

Result 1. If L is targeted, H will join the dictator if D offers $x_H \geq \underline{x}_H \equiv \frac{H(1-F_\epsilon(H+d+w))}{F_\epsilon(H+d+w)}$, otherwise he will not join the conflict. If H is targeted, L will join the dictator if D offers $x_L \geq \underline{x}_L \equiv \frac{(H-w)(1-F_\epsilon(H+d-w))}{F_\epsilon(H+d-w)}$ otherwise he will not join the conflict.

Result 2. If L is targeted, the maximum offer D is willing to offer H is $\overline{x}_H \equiv \frac{(2H+d-w)(F_\epsilon(H+d+w)-F_\epsilon(d+w))}{F_\epsilon(H+d+w)}$. If H is targeted, the maximum offer D is willing to offer L is $\overline{x}_L \equiv \frac{(2H+d)(F_\epsilon(H+d-w)-F_\epsilon(d))}{F_\epsilon(H+d-w)}$.

Proof. Assume the dictator is strong ($d > H - w$) and all possible conflicts are uncertain ($a > H + d + w$).

The dictator targets L and makes an offer to H : $x_H \in [0, 2H + d - w]$

H can join a coalition with the dictator, the target L , or stay out of the conflict.

$$U_H(\text{join}D) = F_\epsilon(H + d + w) (H + x_H)$$

$$U_H(\text{join}L) = (1 - F_\epsilon(d + w - H)) (H)$$

$$U_H(\neg\text{join}) = H$$

Note not joining strictly dominates joining L due to assumed uncertainty

$$\text{Join } D \text{ if } F_\epsilon(H + d + w) (H + x_H) > H$$

$$x_H > \frac{H(1-F_\epsilon(H+d+w))}{F_\epsilon(H+d+w)} \text{ denote this minimal offer } H \text{ is willing to accept as } \underline{x}_H$$

$$BR_H: \text{ join } D \text{ if } x_H \geq \underline{x}_H, \text{ else } \neg\text{join}$$

Given this response function, what offer will the dictator make?

$$U_D(x_H) = F_\epsilon(H + d + w) (2H + d - w - x_H) \text{ if } x_H > \underline{x}_H \text{ (the offer is sufficient)}$$

$$U_D(x_H) = F_\epsilon(d + w)(2H + d - w) \text{ if } x_H < \underline{x}_H \text{ (the offer is insufficient)}$$

The dictator prefers to make a sufficient offer if $F_\epsilon(H + d + w) (2H + d - w - x_H) > F_\epsilon(d + w)(2H + d - w)$

$x_H < \frac{(2H+d-w)(F_\epsilon(H+d+w)-F_\epsilon(d+w))}{F_\epsilon(H+d+w)}$ denote this maximal offer the dictator is willing to make as $\overline{x_H}$

Lemma 1. *Given the dictator's maximal willingness to pay is greater than the elite's minimal offer to join, the dictator will offer the elite's minimum sufficient offer.*

Proof. Assume $\overline{x_H} \geq x_H$. Recall H will join the dictator's coalition for all offers $x_H \geq \underline{x_H}$.

Would the dictator ever make an offer greater than $\underline{x_H}$?

$$U_D(\underline{x_H}) = F_\epsilon(H+d+w)(2H+d-w-\underline{x_H})$$

$$U_D(\underline{x_H} + \eta) = F_\epsilon(H+d+w)(2H+d-w-(\underline{x_H} + \eta)) \text{ where } \eta > 0$$

$$F_\epsilon(H+d+w)(2H+d-w-\underline{x_H}) > F_\epsilon(H+d+w)(2H+d-w-(\underline{x_H} + \eta))$$

$$2H+d-w-\underline{x_H} > 2H+d-w-(\underline{x_H} + \eta)$$

$0 > -\eta$ this is true by definition.

Therefore if the dictator is willing to make a sufficient offer, he will only ever make the minimal sufficient offer to get H to join him. \square

BR_D : offer $\underline{x_H}$ if $\overline{x_H} \geq \underline{x_H}$, else indifferent among all $x_H < \overline{x_H}$

Using $\epsilon \sim U[-a, a]$, $\overline{x_H} \geq \underline{x_H}$ if $a \leq 3H + 2d$

L targeted subgame: If $a \in (H + d + w, 3H + 2d]$, D offer $\underline{x_H}$, H join D.

If $a > 3H + 2d$, D offer any $x_H < \underline{x_H}$, H doesn't join the conflict.

The dictator targets H and makes an offer to $Lx_L \in [0, 2H + d]$

L can join a coalition with the dictator, join with the targeted H, or stay out of the conflict.

$$U_L(\text{join}D) = F_\epsilon(H+d-w)(H-w+x_L)$$

$$U_L(\text{join}H) = (1 - F_\epsilon(d+w-H))(H-w)$$

$$U_L(\neg\text{join}) = H-w$$

Note not joining dominates joining a coalition with H under assumed uncertainty.

Join D if $F_\epsilon(H+d-w)(H-w+x_L) > H-w$

$x_L > \frac{(H-w)(1-F_c(H+d-w))}{F_c(H+d-w)}$ denote this minimal offer L is willing to accept to join the dictator as \underline{x}_L

BR_L : join D if $x_L \geq \underline{x}_L$ else \neg join

Given this expected response function, what will the dictator offer?

$U_D(x_L) = F_c(H+d-w)(2H+d-x_L)$ if $x_L > \underline{x}_L$ (the offer is sufficient)

$U_D(x_L) = F_c(d)(2H+d)$ if $x_L < \underline{x}_L$ (the offer is insufficient)

Make a sufficient offer if $F_c(H+d-w)(2H+d-x_L) > F_c(d)(2H+d)$

$x_L < \frac{(2H+d)(F_c(H+d-w)-F_c(d))}{F_c(H+d-w)}$ denote this maximal offer the dictator is willing to make as \overline{x}_L

Lemma 2. *Given the dictator's maximal willingness to pay is greater than the elite's minimal offer to join, the dictator will offer the elite's minimum sufficient offer.*

Proof. Assume $\overline{x}_L \geq \underline{x}_L$. Recall L will join the dictator's coalition for all offers $x_L \geq \underline{x}_L$.

Would the dictator ever make an offer greater than \underline{x}_L ?

$U_D(\underline{x}_L) = F_c(H+d-w)(2H+d-\underline{x}_L)$

$U_D(\underline{x}_L + \eta) = F_c(H+d-w)(2H+d-(\underline{x}_L + \eta))$ where $\eta > 0$

$F_c(H+d-w)(2H+d-\underline{x}_L) > F_c(H+d-w)(2H+d-(\underline{x}_L + \eta))$

$2H+d-\underline{x}_L > 2H+d-(\underline{x}_L + \eta)$

$0 > -\eta$ this is true by definition.

Therefore if the dictator is willing to make a sufficient offer, he will only ever make the minimal sufficient offer to get L to join him. □

BR_D : offer \underline{x}_L if $\overline{x}_L \geq \underline{x}_L$, else indifferent among all $x_L < \underline{x}_L$

Using $\epsilon \sim U[-a, a]$, $\overline{x}_L \geq \underline{x}_L$ if $a < 3H + 2d - w$

H targeted subgame: if $a \in (H+d+w, 3H+2d-w]$, D offers \underline{x}_L , L joins D .

If $a > 3H + 2d - w$, D offers any $x_L < \underline{x}_L$, $L \neg$ join. □

Targeting Behavior

Proposition 1. *If the dictator's advantage relative to the elites and uncertainty is sufficiently high ($d > a + w - 3H$), D will target L and make H a sufficient offer \underline{x}_H such that H joins the dictator's coalition. If the dictator's relative advantage is sufficiently low ($d < a + w - 3H$), the dictator will choose no target and no conflict will occur.*

Proof. $a \in (H + d + w, 3H + 2d - w)$ both coalitions available

$$U_D(\emptyset) = H + d$$

$$U_D(H) = F_\epsilon(H + d - w) (2H + d - \underline{x}_L) \text{ make sufficient minimal offer to L, L joins D}$$

$$U_D(L) = F_\epsilon(H + d + w) (2H + d - w - \underline{x}_H) \text{ make sufficient minimal offer to L, H joins D}$$

Target H vs No Target

$$F_\epsilon(H + d - w) (2H + d - \underline{x}_L) > H + d$$

$$F_\epsilon(H + d - w)(3H + d - w) > 2H + d - w$$

Using $\epsilon \sim U[-a, a]$:

$a < 3H + d - w$ if this holds, targeting H and offering \underline{x}_L is preferred to no target.

Target L vs No Target

$$F_\epsilon(H + d + w) (2H + d - w - \underline{x}_H) > H + d$$

$$F_\epsilon(H + d + w)(3H + d - w) > 2H + d$$

Using $\epsilon \sim U[-a, a]$

$a < 3H + d - w$ if this holds, targeting L and offering \underline{x}_H is preferred to no target.

Target H vs Target L

Targeting H preferred to Targeting L if:

$$F_\epsilon(H + d - w) (2H + d - \underline{x}_L) > F_\epsilon(H + d + w) (2H + d - w - \underline{x}_H)$$

$$F_\epsilon(H + d - w)(3H + d - w) - (H - w) > F_\epsilon(H + d + w)(3H + d - w) - H$$

Using $\epsilon \sim U[-a, a]$:

$a > 3H + d - w$ if this holds, H is preferred to targeting L. So L is a preferred target when
 $a < 3H + d - w$

Equilibrium: if $a \in (H + d + w, 3H + d - w)$, Target L, offer \underline{x}_H to H, H joins D.

If $a \in (3H + d - w, 3H + 2d - w)$, the dictator chooses no target, no conflict occurs

$a \in (3H + 2d - w, 3H + 2d)$ Only a coalition with H is possible, the dictator and L cannot come to a power-sharing agreement.

$$U_D(\emptyset) = H + d$$

$$U_D(H) = F_\epsilon(d)(2H + d) \text{L will not join D}$$

$$U_D(L) = F_\epsilon(H + d + w) (2H + d - w - \underline{x}_H) \text{ make sufficient minimal offer to H, H joins D}$$

Target H vs No target

$$\text{Target H if } F_\epsilon(d)(2H + d) > H + d$$

$a < 2H + d$ If this holds, targeting H is preferred to no target. However, this does not hold in this range of a therefore no target is preferred.

Target L vs No Target

$$F_\epsilon(H + d + w) (2H + d - w - \underline{x}_H) > H + d$$

$$F_\epsilon(H + d + w)(3H + d - w) > 2H + d$$

Using $\epsilon \sim U[-a, a]$

$a < 3H + d - w$ if this holds, targeting L and offering x_H is preferred to no target. This does not hold here, therefore no target no conflict.

Equilibrium: for $a \in (3H + 2d - w, 3H + 2d)$, the dictator will choose no target and no conflict will occur.

$a > 3H + 2d$ no coalitions would form.

$$U_D(\emptyset) = H + d$$

$U_D(H) = F_\epsilon(d)(2H + d)$ L will not join D

$U_D(L) = F_\epsilon(d + w)(2H + d - w)$ H will not join D

Target H vs No target

Target H if $F_\epsilon(d)(2H + d) > H + d$

$a < 2H + d$ If this holds, targeting H is preferred to no target. However, this does not hold in this range of a therefore no target is preferred.

Target L vs No Target

Target L if $F_\epsilon(d + w)(2H + d - w) > H + d$

$a < 2H + d - w$ if this holds, target L is preferred to no target. This does not hold in this range of a , therefore no target is preferred.

Equilibrium: for $a > 3H + 2d$, the dictator will choose no target and no conflict will occur.

□

Round 2: Two Players Remaining

Proposition 2. *If the dictator's advantage relative to the elites and uncertainty is sufficiently high ($d > a + w - 3H$), the dictator will target the remaining regime member. If the dictator's relative advantage is sufficiently low ($d < a + w - 3H$), the dictator will choose no target and no conflict will occur.*

Proof. There are four ways for there to be exactly two players remaining: (1) H and D fought alone (or L joined H and received no transfer); (2) L and D fought alone (or H joined L and received no transfer); (3) a coalition of D and L fought H so now D and L remain with some power transferred to L; (4) a coalition of D and H fought L, leaving D and H with some power transferred to H.

(1) D or H is the dictator with power $2H + d$, option to target L who has power $H - w$
 Target L if $F_\epsilon(H + d + w)(3H + d - w) > 2H + d$
 Using Uniform, target L if $a < 3H + d - w$ else no conflict

(2) D or L is the dictator with power $2H + d - w$, option to target H who has power H
 Target H if $F_\epsilon(H + d - w)(3H + d - w) > 2H + d - w$
 Using Uniform, target H if $a < 3H + d - w$ else no conflict

(3) A coalition between D and L fought against H in the first round, some power $x_L \in [0, 2H + d]$ was transferred to L. Theoretically, the transfer could have been so large that L's second round power is greater than the original dictator's new power, making L the new dictator. Therefore there are two subcases.

(3a) D is still the dictator ($2H + d - x_L > H - w + x_L$)
 Target L if $F_\epsilon(H + d + w - 2x_L)(3H + d - w) > 2H + d - x_L$
 Using uniform target L if $a < 3H + d - w$ else no conflict

(3b) L is now the dictator and can target D ($2H + d - x_L < H - w + x_L$)
 Target D if $F_\epsilon(2x_L - H - d - w)(3H + d - w) > H - w + x_L$
 Using uniform, target D if $a < 3H + d - w$ else no conflict

(4) A coalition between D and H fought against L in the first round, some power $x_H \in [0, 2H + d - w]$ was transferred to H. Theoretically, the transfer could have been so large that H's second round power is greater than the original dictator's new power, making H the new dictator. Therefore there are two subcases.

(4a) D is still the dictator ($2H + d - w - x_H > H + x_H$)
 Target H if $F_\epsilon(H + d - w - 2x_H)(3H + d - w) > 2H + d - w - x_H$
 Using uniform, target H if $a < 3H + d - w$ else no conflict

(4b) H is now the dictator and can target D ($2H + d - w - x_H < H + x_H$)

Target D if $F_\epsilon(2x_H - H - d + w)(3H + d - w) > H + x_H$

Using uniform target D if $a < 3H + d - w$ else no conflict.

□

Round 1

Proposition 3. *If the dictator's advantage relative to uncertainty is sufficiently low ($d < a + w - 3H$), there will be no conflict. The first round join conditions for the elites are the same as three players in round two ($\underline{x}_H, \underline{x}_L$), but the dictator will not initiate any conflict.*

Result 3. *If the dictator's relative advantage is sufficiently high ($d > a + w - 3H$): if L is targeted, H will join the dictator if D offers $x_H \geq \underline{x}'_H \equiv \frac{(a-H-d-w)(a-H-d+w)}{2(H+d+w+a)}$, otherwise he will not join the conflict. If H is targeted, L will join the dictator if D offers $x_L \geq \underline{x}'_L \equiv \frac{(a-H-d-w)(a-H-d+w)}{2(a+H+d-w)}$, otherwise he will not join the conflict. In this parameter range, $\underline{x}_H > \underline{x}'_H$ and $\underline{x}_L > \underline{x}'_L$.*

Proof. Assume L is targeted.

$$a < 3H + d - w$$

- D makes H a low or insufficient offer

$$U_H(\text{join } D) = F_\epsilon(H + d + w)(1 - F_\epsilon(H + d - w))(3H + d - w) \text{ targeted in the second round}$$

$$U_H(\text{join } L) = (1 - F_\epsilon(d + w - H))(1 - F_\epsilon(H + d - w))(3H + d - w)$$

$$U_H(\neg \text{join}) = (1 - F_\epsilon(H + d - w))(3H + d - w), H \text{ is targeted in round 2}$$

Don't join dominates both coalition options. BR_H is don't join.

- D makes H a sufficient offer

$$U_H(\text{join } D) = F_\epsilon(H + d + w)(1 - F_\epsilon(H + d - w - 2x'_H))(3H + d - w) \text{ targeted in the second round}$$

$$U_H(\text{join } L) = (1 - F_\epsilon(d + w - H))(1 - F_\epsilon(H + d - w))(3H + d - w)$$

$$U_H(\neg \text{join}) = (1 - F_\epsilon(H + d - w))(3H + d - w) H \text{ is targeted in round 2}$$

Note not joining dominates joining L

Join D if $F_\epsilon(H+d+w)(1 - F_\epsilon(H + d - w - 2x'_H))(3H+d-w) > (1 - F_\epsilon(H + d - w))(3H + d - w)$

$x'_H > \frac{(a-H-d-w)(a-H-d+w)}{2(H+d+w+a)}$ denote this minimal offer that H is willing to accept as $\underline{x'_H}$. If the dictator's offer meets this constraint, H will join him in a coalition in the first round. because the join threshold is the same and the ultimate expected utilities of the dictator and H are the same regardless of who becomes the dictator, it doesn't matter whether the transfer is large enough for H to become the dictator

BR_H join D if $x'_H > \underline{x'_H}$ else don't join

- D makes H a high offer, so high that if they win H will become the dictator

$U_H(\text{join}D) = F_\epsilon(H + d + w)F_\epsilon(2x'_H - H - d + w)(3H + d - w)$ targets remaining player in the second round

$U_H(\text{join}L) = (1 - F_\epsilon(d + w - H))(1 - F_\epsilon(H + d - w))(3H + d - w)$

$U_H(\neg\text{join}) = (1 - F_\epsilon(H + d - w))(3H + d - w)$ H is targeted in round 2

Note not joining dominates joining L

Join D if $F_\epsilon(H+d+w)F_\epsilon(2x'_H - H - d + w)(3H+d-w) > (1 - F_\epsilon(H + d - w))(3H+d-w)$
 $x'_H > \frac{(a-H-d-w)(a-H-d+w)}{2(H+d+w+a)}$ denote this minimal offer that H is willing to accept as $\underline{x'_H}$. If the

dictator's offer meets this constraint, H will join him in a coalition in the first round. Note this is the same constraint as above... whether or not H becomes the dictator in the second round will ultimately depend on a .

$$a > 3H + d - w$$

- D makes a low or sufficient offer

$U_H(\text{join}D) = F_\epsilon(H + d + w)(H)$

$U_H(\text{join}L) = (1 - F_\epsilon(d + w - H))(H)$

$$U_H(\neg\text{join}) = H$$

Not joining dominates both coalition options. $BR_H \neg \text{join}$

- D makes H a sufficiently high offer

$$U_H(\text{join}D) = F_\epsilon(H + d + w)(H + x_H)$$

$$U_H(\text{join}L) = (1 - F_\epsilon(d + w - H))(H)$$

$$U_H(\neg\text{join}) = H$$

Note not joining dominates joining L Same as third round: join D if $x_H > \frac{H(1-F_\epsilon(H+d+w))}{F_\epsilon(H+d+w)}$

denoted \underline{x}_H

BR_H join D if $x_H > \underline{x}_H$ else don't join

□

Proof. Assume H is targeted.

$$a < 3H + d - w$$

- D makes L a low or insufficient offer

$$U_L(\text{join}D) = F_\epsilon(H + d - w)(1 - F_\epsilon(H + d + w))(3H + d - w)$$

$$U_L(\text{join}H) = (1 - (F_\epsilon(d + w - H))(1 - F_\epsilon(H + d + w))(3H + d - w)$$

$$U_L(\neg\text{join}) = (1 - F_\epsilon(H + d + w))(3H + d - w)$$

Note not joining either coalition is dominant strategy BR_L : don't join

- D makes L a sufficient offer but not high enough to make L the second round dictator

$$U_L(\text{join}D) = F_\epsilon(H + d - w)(1 - F_\epsilon(H + d + w - 2x'_L))(3H + d - w)$$

$$U_L(\text{join}H) = (1 - (F_\epsilon(d + w - H))(1 - F_\epsilon(H + d + w))(3H + d - w)$$

$$U_L(\neg\text{join}) = (1 - F_\epsilon(H + d + w))(3H + d - w)$$

Note not joining dominates joining H

Join D if $F_\epsilon(H+d-w)(1 - F_\epsilon(H + d + w - 2x'_L))(3H+d-w) > (1 - F_\epsilon(H + d + w))(3H + d - w)$

$x'_L > \frac{(a-d-H-w)(a-d-H+w)}{2(a+d+H-w)}$ denote this minimal offer L is willing to accept as \underline{x}'_L

BR_L : if $x'_L > \underline{x}'_L$ join D, else don't join

confirm that x'_L will not make L the dictator... same as above, the threshold is the same and the expected utilities of L and D will be the same, so whether or not L becomes the dictator doesn't matter

- D makes L a high enough offer to make L the second round dictator

$$U_L(\text{join}D) = F_\epsilon(H + d - w)F_\epsilon(2x'_L - H - d - w)(3H + d - w)$$

$$U_L(\text{join}H) = (1 - F_\epsilon(d + w - H))(1 - F_\epsilon(H + d + w))(3H + d - w)$$

$$U_L(\neg\text{join}) = (1 - F_\epsilon(H + d + w))(3H + d - w)$$

Note not joining dominates joining H

$$\text{Join D if } F_\epsilon(H + d - w)F_\epsilon(2x'_L - H - d - w)(3H + d - w) > (1 - F_\epsilon(H + d + w))(3H + d - w)$$

$$x'_L > \frac{(a-d-H-w)(a-d-H+w)}{2(a+d+H-w)} \text{ denote this minimal offer L is willing to accept as } \underline{x}'_L \text{ note this is}$$

the same as the previous constraint

$$BR_L: \text{ if } x'_L > \underline{x}'_L \text{ join D, else don't join}$$

$$a > 3H + d - w$$

- D makes L a low or insufficient offer

$$U_L(\text{join}D) = F_\epsilon(H + d - w)(H - w)$$

$$U_L(\text{join}H) = (1 - F_\epsilon(d + w - H))(H - w)$$

$$U_L(\neg\text{join}) = H - w$$

don't join dominates both coalition options as conflicts are uncertain

- D makes L a sufficient offer

$$U_L(\text{join}D) = F_\epsilon(H + d - w)(H - w + x_L)$$

$$U_L(\text{join}H) = (1 - F_\epsilon(d + w - H))(H - w)$$

$$U_L(\neg\text{join}) = H - w$$

Note not joining dominates joining H

$$\text{Join D if } F_\epsilon(H + d - w)(H - w + x_L) > H - w$$

$x_L > \frac{(H-w)(1-F_\epsilon(H+d-w))}{F_\epsilon(H+d-w)}$ denote this minimal offer L will accept as \underline{x}_L note this is the same as the third round constraint.

BR_L : join D if $x_L > \underline{x}_L$ else don't join □

Proposition 4. *Assume the dictator's relative advantage is sufficiently large ($d > a + w - 3H$). Offering a minimal sufficient transfer to induce a coalition $(\underline{x}'_H, \underline{x}'_L)$ dominates targeting an elite and failing to form a coalition. When uncertainty is sufficiently low, targeting an elite and making a minimal sufficient offer dominates no conflict initiation.*

Proof. Targeting Decision in Rd 1 given best responses and expected round 2 behavior

if $a < 3H + d - w$ round 2 conflict will occur

(1) Target L make H low or no offer $U_D(L) = F_\epsilon(d+w)F_\epsilon(H+d-w)(3H+d-w)$ H does not join, if dictator wins target H rd 2

(2) target L make H a sufficient offer

$U_D(L, \underline{x}'_H) = F_\epsilon(H+d+w)F_\epsilon(H+d-w-2x'_H)(3H+d-w)$ H joins D, D targets H rd 2

(3) target L make H a huge offer so H will be 2nd round dictator

$U_D(L, \underline{x}'_H) = F_\epsilon(H+d+w)(1-F_\epsilon(2x'_H+w-H-d))(3H+d-w)$, H joins D, H becomes rd2 dictator and targets D. note that this is equivalent to option (2) using the Uniform distribution as the minimal offers are the same

(4) target H make L low or no offer

$U_D(H) = F_\epsilon(d)F_\epsilon(H+d+w)(3H+d-w)$ L does not join, if dictator wins target L rd 2

(5) target H make L sufficient offer

$U_D(H, \underline{x}'_L) = F_\epsilon(H+d-w)F_\epsilon(H+d+w-2x'_L)(3H+d-w)$ L joins D, D targets L rd 2

(6) target H make L a huge offer so L will be second round dictator

$U_D(H, \underline{x}'_L) = F_\epsilon(H+d-w)(1-F_\epsilon(2x'_L-w-H-d))(3H+d-w)$, L joins D, L becomes rd 2 dictator and targets D. note this is equivalent to (5) using the uniform as the minimum offers are the same

(7) no round 1 conflict

$U_D(\emptyset) = F_\epsilon(H + d + w)(2H + d - w - \underline{x}_H)$ L is targeted in round 2, D offers \underline{x}_H and H joins
D

Compare $(L, 0)$ and $(H, 0)$

$$F_\epsilon(d + w)F_\epsilon(H + d - w)(3H + d - w) > F_\epsilon(d)F_\epsilon(H + d + w)(3H + d - w)$$

using Uniform $Hw - w^2 > aw + dw$

$H - d - w > a$ this does not hold as $a > H + d + w$ to ensure all conflicts uncertain. Therefore
 $(H, 0)$ dominates $(L, 0)$.

Compare (H, \underline{x}'_L) and (L, \underline{x}'_H)

$$F_\epsilon(H + d - w)F_\epsilon(H + d + w - 2\underline{x}'_L)(3H + d - w) > F_\epsilon(H + d + w)F_\epsilon(H + d - w - 2\underline{x}'_H)(3H + d - w)$$

Using uniform and substituting in the offers

$$(H + d + w + a)(H + d - w + a) - (a - d - H - w)(a - d - H + w) > (H + d + w + a)(H + d - w + a) - (a - H - d - w)(a - H - d + w)$$

indifferent

Compare (L, \underline{x}'_H) and $(H, 0)$

$$F_\epsilon(H + d + w)F_\epsilon(H + d - w - 2\underline{x}'_H)(3H + d - w) > F_\epsilon(d)F_\epsilon(H + d + w)(3H + d - w)$$

$$F_\epsilon(H + d - w - 2\underline{x}'_H) > F_\epsilon(d)$$

$$H - w > 2\underline{x}'_H$$

$$H - w > \frac{(a - H - d - w)(a - H - d + w)}{(H + d + w + a)}$$

$$\text{Note } \lim_{a \rightarrow H + d + w} H - w - \frac{(a - H - d - w)(a - H - d + w)}{(H + d + w + a)} = \frac{d(H - w)}{2H + d} > 0$$

$$\text{and } \lim_{a \rightarrow 3H + d - w} H - w - \frac{(a - H - d - w)(a - H - d + w)}{(H + d + w + a)} = H - w > 0$$

Check that $(L, \underline{x}'_H) - (H, 0)$ does not cross zero for $a \in (H + d + w, 3H + d - w)$

$$\text{Roots: } \frac{1}{2}(3H + 2d - w \pm \sqrt{8Hd + 9H^2 - 8dw - 6Hw + w^2})$$

$\frac{1}{2}(3H + 2d - w - \sqrt{8Hd + 9H^2 - 8dw - 6Hw + w^2}) < H + d + w$ the lower root is less than the lower case bound of uncertainty

$\frac{1}{2}(3H + 2d - w + \sqrt{8Hd + 9H^2 - 8dw - 6Hw + w^2}) > 3H + d - w$ the upper root is greater than the upper case bound

Thus the relative utility function does not cross zero inside this case space, thus $(L, \underline{x'_H}) > (H, 0)$ for all $a \in (H + d + w, 3H + d - w)$

So the dictator is indifferent between each of the target/offers, but a target with an offer dominates targeting without making an offer and forming a coalition in this range of uncertainty. How does this compare to no first round target?

Compare (L, x'_H) and (\emptyset)

Target an elite and make a coalition offer is preferred to no target in the first round if:

$$F_\epsilon(H + d + w)F_\epsilon(H + d - w - 2x'_H)(3H + d - w) - F_\epsilon(H + d + w)(2H + d - w - x_H) > 0$$

Substituting in minimal offers and using Uniform,

$$\lim_{a \rightarrow 3H+d-w} \frac{2a(3H+d-w)(2H+2d-w) - (3H+d-w)(H+d-w)(H+d+w) + a^2(3w-5H-3d)}{4a^2} = \frac{H(w-H)}{3H+d-w} < 0$$

by definition of H and w . Thus as a approaches its upper bound in this range, no target is preferred to an elite target and coalition

$$\lim_{a \rightarrow H+d+w} \frac{2a(3H+d-w)(2H+2d-w) - (3H+d-w)(H+d-w)(H+d+w) + a^2(3w-5H-3d)}{4a^2} = \frac{(H-w)(H+d-w)}{H+d+w} > 0$$

by definition of H and w . Thus as a approaches its lower bound in this range, though conflict outcomes are still uncertain, targeting an elite and making a sufficient offer to form a coalition is preferred to no target. There exists a range of parameters for which the dictator prefers initiating conflict and forming an elite coalition.

$$a > 3H + d - w \text{ no conflict round 2}$$

No target: $U_D(\emptyset) = H + d$

Target L , no or low offer to H : $F_\epsilon(d + w)(2H + d - w)$

Target L, sufficient offer to H : $F_e(H + d + w) (2H + d - w - \underline{x}_H)$

Target H, low or no offer: $F_e(d)(2H + d - w) (2H + d - w - x_H)$

Target H, sufficient offer to L : $F_e(A + d - w) (2H + d - \underline{x}_L)$

No target preferred to target L with offer ($a > 3H + d - w$), No target preferred to with

offer, No target preferred to target L no offer ($a > 2H + d - w$), No target preferred to target

H no offer ($a > 2H + d$), no target preferred to H with offer ($a > 3H + d - w$)

no target

□

Appendix C: Rubinstein Bargaining

Set up: the dictator chooses the bargaining order (H first or L first)

the dictator bargains with first elite with alternating offer protocol with δ probability that bargaining continues; if they reach an agreement, they form coalition and fight the other elite, sharing power post-conflict according to the bargain

If they fail to reach an agreement, the dictator bargains with the next elite to form a coalition against the first, sharing power according to the agreement

If they fail to reach an agreement, the dictator can fight either elite alone or not start a conflict and maintain his power

All conflicts are subject to uniform noise contest

General Bargaining SPNE

Denote the dictator's outside option Ω_D and the elite's outside option Ω_E . Regardless of the history, after any rejection bargaining continues with common probability $\delta \in (0, 1)$. The dictator makes the first offer.

Let m_E and M_E be the infimum and supremum of equilibrium payoffs to the elite when he is the proposer. Let m_D and M_D be the infimum and supremum of equilibrium payoffs to the dictator when he is the proposer. The following inequalities hold:

1. $m_E \geq 1 - (\delta M_D + (1 - \delta)\Omega_D)$

2. $M_E \leq 1 - (\delta m_D + (1 - \delta)\Omega_D)$

3. $m_D \geq 1 - (\delta M_E + (1 - \delta)\Omega_E)$

4. $M_D \leq 1 - (\delta m_E + (1 - \delta)\Omega_E)$

In equilibrium, the dictator must accept an offer x where $x = (\delta M_D + (1 - \delta)\Omega_D)$ as that is the most that he could get from refusing (inequality 1). It follows that the elite cannot get less than w where $w = 1 - (\delta M_D + (1 - \delta)\Omega_D)$ because he can get a guaranteed w by making it his opening demand.

Similarly, in equilibrium, the dictator must get at least y for each $y = (\delta m_D + (1 - \delta)\Omega_D)$ because y is guaranteed if the dictator rejects the elite's opening proposal, so the elite can get at most $1 - y$ (inequality 2).

When the dictator is the proposer, the elite must accept an offer x' where $x' = (\delta M_E + (1 - \delta)\Omega_E)$, the most he could get from refusing. Thus the dictator cannot get less than $1 - (\delta M_E + (1 - \delta)\Omega_E)$, which he is guaranteed if he makes x' his opening proposal (inequality 3)

The elite must get at least y' for each $y' = (\delta m_E + (1 - \delta)\Omega_E)$ as that is guaranteed if the elite rejects the dictator's proposal. Thus the dictator can get at most $1 - y'$ (inequality 4).

Rearranging these inequalities, we see that

$$M_E \leq 1 - \delta m_D - (1 - \delta)\Omega_D$$

$$m_E \geq 1 - \delta M_D - (1 - \delta)\Omega_D$$

so if $m_D = M_D$, then $m_E = M_E$

Further,

$$m_D \geq 1 - \delta M_E - (1 - \delta)\Omega_E$$

$$M_D \leq 1 - \delta m_E - (1 - \delta)\Omega_E$$

So if $m_E = M_E$ then $m_D = M_D$

but how do we know that this is necessarily the case?

Proof by contradiction (to show that $m_D = M_D$)

Assume $m_D < M_D$

From above, we know that $M_D \leq 1 - \delta m_E - (1 - \delta)\Omega_E$ and $m_D \geq 1 - \delta M_E - (1 - \delta)\Omega_E$

$m_D - M_D \geq 1 - \delta M_E - (1 - \delta)\Omega_E - 1 + \delta m_E + (1 - \delta)\Omega_E$ subtracting the lesser from the greater maintains the inequality

$$m_D - M_D \geq \delta(m_E - M_E)$$

Further,

$$M_E \leq 1 - \delta m_D - (1 - \delta)\Omega_D \text{ and } m_E \geq 1 - \delta M_D - (1 - \delta)\Omega_D$$

Thus $m_E - M_E \geq 1 - \delta M_D - (1 - \delta)\Omega_D - (1 - \delta m_D - (1 - \delta)\Omega_D)$ to maintain the inequality

$$m_E - M_E \geq \delta(m_D - M_D)$$

$$\frac{m_E - M_E}{\delta} \geq m_D - M_D$$

Combining the above,

$$\frac{m_E - M_E}{\delta} \geq m_D - M_D \geq \delta(m_E - M_E)$$

By hypothesis, $m_D - M_D < 0$ and as $\delta \in (0, 1)$ by definition, $m_E - M_E < 0$

$$\frac{m_E - M_E}{\delta} \geq \delta(m_E - M_E)$$

$$m_E - M_E \geq \delta^2(m_E - M_E)$$

This is a contradiction as $\delta \in (0, 1)$ and $m_E - M_E < 0$ (multiplying $m_E - M_E$ by a positive number less than one will make it less negative and therefore larger)

Therefore it must be that $m_D \geq M_D$

$m_D > M_D$ also does not hold as the infimum cannot be greater than the supremum by definition

If $m_D \not\leq M_D$ and $m_D \not\geq M_D$, it must be that $m_D = M_D$ and, from above, this implies that $m_E = M_E$

Therefore $m_D = M_D$ and $m_E = M_E$. The subgame perfect equilibrium must be unique.

Single Conflict

First, cases based on the dictator's final outside option: if bargaining with both elites fails, what will the dictator do?

if $a \in (H + d + w, 2H - w)$, target L alone

if $a \in (2H - w, 2H + d)$ target H alone

if $a > 2H + d$ no conflict

Case 1 $a \in (H + d + w, 2H - w)$, outside option is target L alone

If the dictator fails to reach an agreement with either elite, he will target L alone and his expected utility is $F_\epsilon(d + w)(2H + d - w)$

Subgame: Bargain with H first, L second

second bargain with L: dictator proposes x_L to keep $1 - x_L$; elite proposes y_L to keep $1 - y_L$; these are proportions of the benefit from defeating the other elite H

$$U_D(\text{accept}) = F_\epsilon(H + d - w)(H + d + Hy_L)$$

$$U_D(\text{reject}) = \delta(F_\epsilon(H + d - w)(H + d + H(1 - x_L))) + (1 - \delta)(F_\epsilon(d + w)(2H + d - w))$$

$$U_L(\text{accept}) = F_\epsilon(H + d - w)(H - w + Hx_L)$$

$$U_L(\text{reject}) = \delta(F_\epsilon(H + d - w)(H - w + H(1 - y_L))) + (1 - \delta)((1 - F_\epsilon(d + w))(2H + d - w))$$

$$x = (2 * a * (\delta * (((H + d - w + a)/(2 * a)) * (H - w + H * (1 - y)))) + (1 - \delta) * (1 - (d + w + a)/(2 * a))) * (2 * H + d - w) - (((H - w)(a + d + H - w)/(2a)))/(H(a + d + H - w))$$

$$y = ((\delta * (((H + d - w + a)/(2 * a)) * (H + d + H * (1 - x)))) + (1 - \delta) * ((d + w + a)/(2 * a)) * (2 * H + d - w) - ((d + H)(a + d + H - w))/(2a)) * 2 * a)/(H(a + d + H - w))$$

$$x_L^* = \frac{(-d^2 - 3dH - H^2 + dw + (H(d + 2H) - (d + 4H)w + w^2)\delta + a(d + H + w\delta))}{(H(a + d + H - w)(1 + \delta))}$$

$$y_L^* = \frac{(-1 + \delta)(H^2(1 - 2\delta) + Hw(-3 + \delta) - 4dH\delta - (d - w)(w + d\delta) + a(-H + w + d\delta))}{(1 + H(-a + d + H - w)\delta^2)}$$

Leave CDFs as CDFs

$$x_L^* = \frac{2H + d - w + F_\epsilon(H + d - w)(\delta(2H + d) + w - H) - F_\epsilon(d + w)(1 + \delta)(2H + d - w)}{H(1 + \delta)F_\epsilon(H + d - w)}$$

$$y_L^* = \frac{F_\epsilon(d + w)(1 + \delta)(2H + d - w) - \delta(2H + d - w) + F_\epsilon(H + d - w)(\delta(2H - w) - H - d)}{H(1 + \delta)F_\epsilon(H + d - w)}$$

$$U_D = F_\epsilon(H + d - w)(H + d + H(1 - x_L^*))$$

$$= \frac{F_\epsilon(d + w)(2H + d - w)(1 + \delta) + F_\epsilon(H + d - w)(3H + d - w) - (2H + d - w)}{1 + \delta}$$

$$U_L = F_\epsilon(H + d - w)(H - w + Hx_L^*)$$

$$= \frac{F_\epsilon(H + d - w)\delta(3H + d - w) + (2H + d - w) - F_\epsilon(d + w)(2H + d - w)(1 - \delta)}{1 + \delta}$$

First Bargain with H: dictator proposes x_H to keep $1 - x_H$; elite proposes y_H to keep $1 - y_H$; now the outside options take the dictator's coalition with L into account

$$U_D(\text{accept}) = F_\epsilon(H + d + w)(H + d + (H - w)y_H)$$

$$U_D(\text{reject}) = \delta F_\epsilon(H + d + w)(H + d + (H - w)(1 - x_H)) + (1 - \delta) \left(\frac{F_\epsilon(d + w)(2H + d - w)(1 + \delta) + F_\epsilon(H + d - w)(3H + d - w) - (2H + d - w)}{1 + \delta} \right)$$

$$U_H(\text{accept}) = F_\epsilon(H + d + w)(H + (H - w)x_H)$$

$$U_H(\text{reject}) = \delta F_\epsilon(H + d + w)(H + (H - w)(1 - y_H)) + (1 - \delta)(1 - F_\epsilon(H + d - w))(3H + d - w)$$

(note if H doesn't come to an agreement with the dictator, he will fight D and L together with a chance to beat them both)

$$y_H^* = \frac{F_\epsilon(H + d - w)(3H + d - w)(1 + \delta + \delta^*) + F_\epsilon(d + w)(1 + \delta)(2H + d - w) - F_\epsilon(H + d + w)(1 + \delta)(H + d + \delta(w - 2H)) - (2H + d - w) - \delta(3H + d - w) - \delta^2}{F_\epsilon(H + d + w)(H - w)(1 + \delta)^2}$$

$$x_H^* = \frac{F_\epsilon(H + d + w)(\delta(H + d - w) + \delta^2(2H + d - w) - H) + (3H + d - w) + \delta(5H + 2d - 2w) - F_\epsilon(H + d - w)(3H + d - w)(1 + 2\delta) - F_\epsilon(d + w)\delta(1 + \delta)(2H + d - w)}{F_\epsilon(H + d + w)(H - w)(1 + \delta)^2}$$

$$U_D = F_\epsilon(H + d + w)(H + d + (H - w)(1 - x_H^*))$$

$$= \frac{F_\epsilon(H + d + w)(1 + \delta)(3H + d - w) - (3H + d - w) - \delta(5H + 2d - 2w) + F_\epsilon(d + w)(2H + d - w)\delta(1 + \delta) + F_\epsilon(H + d - w)(3H + d - w)(1 + 2\delta)}{(1 + \delta)^2}$$

$$U_H = F_\epsilon(H + d + w)(H + (H - w)x_H^*)$$

$$= \frac{F_\epsilon(H + d + w)(3H + d - w)\delta(1 + \delta) - F_\epsilon(H + d - w)(3H + d - w)(1 + 2\delta) - F_\epsilon(d + w)(2H + d - w)\delta(1 + \delta) + (3H + d - w) + \delta(5H + 2d - 2w)}{(1 + \delta)^2}$$

Subgame: Bargain with L first, H second

Backwards induction start with H bargain (outside option is still target L alone)

$$U_D(\text{accept}) = F_\epsilon(H + d + w)(H + d + (H - w)y_H)$$

$$U_D(\text{reject}) = \delta(F_\epsilon(H + d + w)(H + d + (H - w)(1 - x_H))) + (1 - \delta)(F_\epsilon(d + w)(2H + d - w))$$

$$U_H(\text{accept}) = F_\epsilon(H + d + w)(H + (H - w)x_H)$$

$$U_H(\text{reject}) = \delta(F_\epsilon(H + d + w)(H + (H - w)(1 - y_H))) + (1 - \delta)(H)$$

if they don't reach an agreement, dictator fights L and H keeps his power

$$y_H^* = \frac{F_\epsilon(\delta(2H-w)-d-H)+F_\epsilon(d+w)(2H+d-w)-\delta H}{F_\epsilon(H+d+w)(H-w)(1+\delta)}$$

$$x_H^* = \frac{F_\epsilon(H+d+w)(\delta(2H+d-w)-H)-F_\epsilon(d+w)(\delta(2H+d-w))+H}{F_\epsilon(H+d+w)(H-w)(1+\delta)}$$

$$U_D = F_\epsilon(H + d + w)(H + d + (H - w)(1 - x_H^*))$$

$$= \frac{F_\epsilon(H+d+w)(3H+d-w)-H+F_\epsilon(d+w)(2H+d-w)}{1+\delta}$$

$$U_H = F_\epsilon(H + d + w)(H + (H - w)x_H^*)$$

$$= \frac{F_\epsilon(H+d+w)(\delta(3H+d-w))-F_\epsilon(d+w)\delta(2H+d-w)+H}{1+\delta}$$

First bargain with L

$$U_D(\text{accept}) = F_\epsilon(H + d - w)(H + d + H(y_L))$$

$$U_D(\text{reject}) = \delta F_\epsilon(H + d - w)(H + d + H(1 - x_L)) + (1 - \delta) \left(\frac{F_\epsilon(H+d+w)(3H+d-w)-H+F_\epsilon(d+w)(2H+d-w)}{1+\delta} \right)$$

$$U_L(\text{accept}) = F_\epsilon(H + d - w)(H - w + H(x_L))$$

$$U_L(\text{reject}) = \delta F_\epsilon(H + d - w)(H - w + H(1 - y_L)) + (1 - \delta)((1 - F_\epsilon(H + d + w))(3H + d - w))$$

$$y_L^* = \frac{F_\epsilon(3H+d-w)(1+\delta+\delta^2)+F_\epsilon(H+d-w)(\delta(H-d-w)+\delta^2(2H-w)-d-H)+F_\epsilon(d+w)(\delta(2H+d-w))-H-\delta(3H+d-w)-\delta^2(3H+d-w)}{F_\epsilon(H+d-w)H(1+\delta)^2}$$

$$x_L^* = \frac{F_\epsilon(H+d-w)(\delta(H+d+w)+\delta^2(2H+d)-H+w)+(3H+d-w)+\delta(4H+d-w)-F_\epsilon(H+d+w)(3H+d-w)(1+2\delta)-F_\epsilon(d+w)(\delta^2(2H+d-w))}{F_\epsilon(H+d-w)H(1+\delta)^2}$$

$$U_D = F_\epsilon(H + d - w)(H + d + H(1 - x_L^*))$$

$$= \frac{F_\epsilon(3H+d-w+2\delta(3H+d-w))+F_\epsilon(H+d-w)(1+\delta)(3H+d-w)+F_\epsilon(d+w)\delta^2(2H+d-w)-(3H+d-w)-\delta(4H+d-w)}{(1+\delta)^2}$$

$$U_L = F_\epsilon(H + d - w)(H - w + H(x_L^*))$$

$$= \frac{F_\epsilon(H+d-w)(\delta(3H+d-w)+\delta^2(3H+d-w)+(3H+d-w)+\delta(4H+d-w)-F_\epsilon(H+d+w)(3H+d-w)(1+2\delta)-F_\epsilon(d+w)\delta^2(2H+d-w)}{(1+\delta)^2}$$

Which bargaining order would the dictator choose? (which coalition member does he choose)?

Compare

$$U_D(L, H) = \frac{F_\epsilon(3H+d-w+2\delta(3H+d-w))+F_\epsilon(H+d-w)(1+\delta)(3H+d-w)+F_\epsilon(d+w)\delta^2(2H+d-w)-(3H+d-w)-\delta(4H+d-w)}{(1+\delta)^2}$$

$$U_D(H, L) = \frac{F_\epsilon(H+d+w)(1+\delta)(3H+d-w)-(3H+d-w)-\delta(5H+2d-2w)+F_\epsilon(d+w)(2H+d-w)\delta(1+\delta)+F_\epsilon(H+d-w)(3H+d-w)(1+2\delta)}{(1+\delta)^2}$$

$$U_D(L, H) - U_D(H, L) =$$

$$\frac{F_\epsilon(H+d+w)\delta(3H+d-w)+F_\epsilon(H+d-w)(3H+d-w)(1+\delta)-F_\epsilon(d+w)\delta(2H+d-w)-\delta(3H+d-w)-(2H+d-w)}{(1+\delta)^2}$$

Substituting in the uniform win probabilities, the dictator prefers bargaining with L first in this range of a ($a \in H + d + w, 2H - w$). The dictator will make a coalition with L and they will both fight H

Case 2: $a \in (2H - w, 2H + d)$ H alone is the outside option

If the dictator fails to reach an agreement with either elite, he will target H alone and his expected utility is $F_\epsilon(d)(2H + d)$

Subgame: Bargain with H first, L second

second bargain with L: dictator proposes x_L to keep $1 - x_L$; elite proposes y_L to keep $1 - y_L$; these are proportions of the benefit from defeating the other elite H

$$U_D(\text{accept}) = F_\epsilon(H + d - w)(H + d + Hy_L)$$

$$U_D(\text{reject}) = \delta(F_\epsilon(H + d - w)(H + d + H(1 - x_L))) + (1 - \delta)(F_\epsilon(d)(2H + d))$$

$$U_L(\text{accept}) = F_\epsilon(H + d - w)(H - w + Hx_L)$$

$U_L(\text{reject}) = \delta(F_\epsilon(H + d - w)(H - w + H(1 - y_L))) + (1 - \delta)(H - w)$ If they don't reach an agreement, dictator fights H and L stays out of the conflict

$$x_L^* = \frac{F_\epsilon(H+d-w)(\delta(2H+d)-H+w)-F_\epsilon(d)\delta(2H+d)+H-w}{F_\epsilon(H+d-w)H(1+\delta)}$$

$$y_L^* = \frac{F_\epsilon(H+d-w)(\delta(2H-w)-d-H)+F_\epsilon(d)(2H+d)-\delta(H-w)}{F_\epsilon(H+d-w)H(1+\delta)}$$

$$U_D = \frac{F_\epsilon(H+d-w)(3H+d-w)-(H-w)+F_\epsilon(d)\delta(2H+d)}{1+\delta}$$

$$U_L = \frac{F_\epsilon(H+d-w)\delta(3H+d-w)+(H-w)-F_\epsilon(d)\delta(2H+d)}{1+\delta}$$

first bargain with H: dictator proposes x_H to keep $1 - x_H$; elite proposes y_H to keep $1 - y_H$; these are proportions of the benefit from defeating the other elite L

$$U_D(\text{accept}) = F_\epsilon(H + d + w)(H + d + (H - w)y_H)$$

$$U_D(\text{reject}) = \delta F_\epsilon(H + d + w)(H + d + (H - w)(1 - x_H)) + (1 - \delta) \left(\frac{F_\epsilon(H + d - w)(3H + d - w) - (H - w) + F_\epsilon(d)\delta(2H + d)}{1 + \delta} \right)$$

$$U_H(\text{accept}) = F_\epsilon(H + d + w)(H + (H - w)x_H)$$

$$U_H(\text{reject}) = \delta F_\epsilon(H + d + w)(H + (H - w)(1 - y_H)) + (1 - \delta)(1 - F_\epsilon(H + d - w))(3H + d - w)$$

(note if H doesn't come to an agreement with the dictator, he will fight D and L together with a chance to beat them both)

$$x_H^* = \frac{F_\epsilon(H + d + w)(1 + \delta)(\delta(2H + d - w) - H) - F_\epsilon(H + d - w)(3H + d - w)(1 + 2\delta) - F_\epsilon(d)\delta^2(2H + d) + (3H + d - w) + \delta(4H + d - 2w)}{F_\epsilon(H + d + w)(H - w)(1 + \delta)^2}$$

$$y_H^* = \frac{F_\epsilon(H + d + w)(\delta(H - d - w) + \delta^2(2H - w) - d - H) + F_\epsilon(H + d - w)(3H + d - w)(1 + \delta + \delta^2) + F_\epsilon(d)\delta(2H + d) - H + w - \delta(3H + d - w) - \delta^2(3H + d - w)}{F_\epsilon(H + d + w)(H - w)(1 + \delta)^2}$$

$$U_D = F_\epsilon(H + d + w)(H + d + (H - w)(1 - x_H^*))$$

$$= \frac{F_\epsilon(H + d + w)(1 + \delta)(3H + d - w) + F_\epsilon(H + d - w)(3H + d - w)(1 + 2\delta) + F_\epsilon(d)\delta^2(2H + d) - (3H + d - w) - \delta(4H + d - 2w)}{(1 + \delta)^2}$$

$$U_H = F_\epsilon(H + d + w)(H + (H - w)(x_H^*))$$

$$= \frac{F_\epsilon(H + d + w)(3H + d - w)\delta(1 + \delta) - F_\epsilon(H + d - w)(3H + d - w)(1 + 2\delta) - F_\epsilon(d)\delta^2(2H + d) + 3H + d - w + \delta(4H + d - 2w)}{(1 + \delta)^2}$$

Subgame: Bargain with L first, then H

Second bargain with H: dictator proposes x_H to keep $1 - x_H$; elite proposes y_H to keep $1 - y_H$; these are proportions of the benefit from defeating the other elite L. If they do not reach an agreement, he will target H alone and his expected utility is $F_\epsilon(d)(2H + d)$

$$U_D(\text{accept}) = F_\epsilon(H + d + w)(H + d + (H - w)y_H)$$

$$U_D(\text{reject}) = \delta F_\epsilon(H + d + w)(H + d + (H - w)(1 - x_H)) + (1 - \delta)F_\epsilon(d)(2H + d)$$

$$U_H(\text{accept}) = F_\epsilon(H + d + w)(H + (H - w)x_H)$$

$$U_H(\text{reject}) = \delta F_\epsilon(H + d + w)(H + (H - w)(1 - y_H)) + (1 - \delta)(1 - F_\epsilon(d))(2H + d)$$

if they don't reach an agreement, the dictator will fight H alone

$$y_H^* = \frac{F_\epsilon(H + d + w)(\delta(2H - w) - d - H) + F_\epsilon(d)(1 + \delta)(2H + d) - \delta(2H + d)}{F_\epsilon(H + d + w)(H - w)(1 + \delta)}$$

$$x_H^* = \frac{F_\epsilon(H + d + w)(\delta(2H + d - w) - H) - F_\epsilon(d)(2H + d)(1 + \delta) + 2H + d}{F_\epsilon(H + d + w)(H - w)(1 + \delta)}$$

$$U_D = F_\epsilon(H + d + w)(H + d + (H - w)(1 - x_H^*))$$

$$= \frac{F_\epsilon(H+d+w)(3H+d-w)+F_\epsilon(d)(1+\delta)(2H+d)-(2H+d)}{1+\delta}$$

$$U_H = F_\epsilon(H + d + w)(H + (H - w)(x_H^*))$$

$$= \frac{F_\epsilon(H+d+w)\delta(3H+d-w)-F_\epsilon(d)(1+\delta)(2H+d)+2H+d}{1+\delta}$$

first bargain with L: dictator proposes x_L to keep $1 - x_L$; elite proposes y_L to keep $1 - y_L$; these are proportions of the benefit from defeating the other elite H; if they don't reach an agreement, the dictator will work with H and fight L

$$U_D(\text{accept}) = F_\epsilon(H + d - w)(H + d + Hy_L)$$

$$U_D(\text{reject}) = \delta(F_\epsilon(H+d-w)(H+d+H(1-x_L))) + (1-\delta)\left(\frac{F_\epsilon(H+d+w)(3H+d-w)+F_\epsilon(d)(1+\delta)(2H+d)-(2H+d)}{1+\delta}\right)$$

$$U_L(\text{accept}) = F_\epsilon(H + d - w)(H - w + Hx_L)$$

$$U_L(\text{reject}) = \delta(F_\epsilon(H + d - w)(H - w + H(1 - y_L))) + (1 - \delta)(1 - F_\epsilon(H + d + w))(3H + d - w)$$

$$y_L^* = \frac{F_\epsilon(H+d+w)(1+\delta+\delta^2)(3H+d-w)+F_\epsilon(H+d-w)(\delta^2(2H-w)+\delta(H-d-w)-d-H)+F_\epsilon(d)(1+\delta)(2H+d)+d-2H-\delta(3H+d-w)-\delta^2(3H+d-w)}{F_\epsilon(H+d-w)H(1+\delta)^2}$$

$$x_L^* = \frac{F_\epsilon(H+d-w)(\delta(H+d+w)+\delta^2(2H+d)-H+w)-F_\epsilon(H+d+w)(3H+d-w)(1+2\delta)-F_\epsilon(d)\delta(1+\delta)(2H+d)+3H+d-w+\delta(5H+2d-w)}{F_\epsilon(H+d-w)H(1+\delta)^2}$$

$$U_D = F_\epsilon(H + d - w)(H + d + H(1 - x_L^*))$$

$$= \frac{F_\epsilon(H+d+w)(3H+d-w)(1+2\delta)+F_\epsilon(H+d-w)(3H+d-w)(1+\delta)+F_\epsilon(d)\delta(1+\delta)(2H+d)-(3H+d-w)-\delta(5H+2d-w)}{(1+\delta)^2}$$

$$U_L = F_\epsilon(H + d - w)(H - w + Hx_L^*)$$

$$= \frac{F_\epsilon(H+d-w)\delta(1+\delta)(3H+d-w)-F_\epsilon(H+d+w)(3H+d-w)(1+2\delta)-F_\epsilon(d)\delta(1+\delta)(2H+d)+3H+d-w+\delta(5H+2d-w)}{(1+\delta)^2}$$

Who does the dictator want to bargain with first?

$$U_D(L, H) - U_D(H, L) =$$

$$\frac{F_\epsilon(H+d+w)(3H+d-w)(\delta^2-\delta-1)-F_\epsilon(H+d-w)(3H+d-w)(2+3\delta)-F_\epsilon(d)(2H+d)(1+2\delta)+(3H+d-w)(2+3\delta)}{(1+\delta)^2}$$

Substituting in the uniform win probabilities, this is negative in this subgame range of a ($a \in (2H - w, 2H + d)$). So the dictator prefers to bargain with H first then L. In equilibrium, the dictator forms a coalition with H and the fight L together.

Case 3: $a > 2H + d$ no conflict is dictator's outside option

If the dictator fails to reach an agreement with either elite, he will target not target either elite and there will be no conflict

Subgame: Bargain with H first, L second

second bargain with L: dictator proposes x_L to keep $1 - x_L$; elite proposes y_L to keep $1 - y_L$; these are proportions of the benefit from defeating the other elite H

$$U_D(\text{accept}) = F_\epsilon(H + d - w)(H + d + Hy_L)$$

$U_D(\text{reject}) = \delta(F_\epsilon(H + d - w)(H + d + H(1 - x_L))) + (1 - \delta)(H + d)$ no conflict, he keeps his status quo power

$$U_L(\text{accept}) = F_\epsilon(H + d - w)(H - w + Hx_L)$$

$U_L(\text{reject}) = \delta(F_\epsilon(H + d - w)(H - w + H(1 - y_L))) + (1 - \delta)(H - w)$ If they don't reach an agreement, no conflict keep status quo power

$$y_L^* = \frac{F_\epsilon(H+d-w)(\delta(2H-w)-d-H)-\delta(H-w)+H+d}{F_\epsilon(H+d-w)H(1+\delta)}$$

$$x_L^* = \frac{F_\epsilon(H+d-w)(\delta(2H+d)-H+w)-\delta(H-d)+H-w}{F_\epsilon(H+d-w)H(1+\delta)}$$

$$\begin{aligned} U_D &= F_\epsilon(H + d - w)(H + d + H(1 - x_L^*)) \\ &= \frac{F_\epsilon(H+d-w)(3H+d-w)+\delta(H+d)-(H-w)}{1+\delta} \end{aligned}$$

$$\begin{aligned} U_L &= F_\epsilon(H + d - w)(H - w + Hx_L^*) \\ &= \frac{F_\epsilon(H+d-w)\delta(3H+d-w)-\delta(H+d)+H-w}{1+\delta} \end{aligned}$$

first bargain with H: dictator proposes x_H to keep $1 - x_H$; elite proposes y_H to keep $1 - y_H$; these are proportions of the benefit from defeating the other elite L

$$U_D(\text{accept}) = F_\epsilon(H + d + w)(H + d + (H - w)y_H)$$

$$U_D(\text{reject}) = \delta F_\epsilon(H + d + w)(H + d + (H - w)(1 - x_H)) + (1 - \delta) \left(\frac{F_\epsilon(H + d - w)(3H + d - w) + \delta(H + d) - (H - w)}{1 + \delta} \right)$$

$$U_H(\text{accept}) = F_\epsilon(H + d + w)(H + (H - w)x_H)$$

$$U_H(\text{reject}) = \delta F_\epsilon(H + d + w)(H + (H - w)(1 - y_H)) + (1 - \delta)(1 - F_\epsilon(H + d - w))(3H + d - w)$$

(note if H doesn't come to an agreement with the dictator, he will fight D and L together with a chance to beat them both)

$$y_H^* = \frac{F_\epsilon(H + d + w)(\delta^2(2H - w) + \delta(H - d - w) - d - H) + F_\epsilon(H + d - w)(1 + \delta + \delta^2)(3H + d - w) - \delta^2(3H + d - w) - \delta(2H - w) - (H - w)}{F_\epsilon(H + d + w)(H - w)(1 + \delta)^2}$$

$$x_H^* = \frac{F_\epsilon(H + d + w)(\delta^2(2H + d - w) + \delta(H + d - w) - H) - F_\epsilon(H + d - w)(3H + d - w)(1 + 2\delta) - \delta^2(H + d) + \delta(4H + d - 2w) + 3H + d - w}{F_\epsilon(H + d + w)(H - w)(1 + \delta)^2}$$

$$U_D = F_\epsilon(H + d + w)(H + d + (H - w)(1 - x_H^*))$$

$$= \frac{F_\epsilon(H + d + w)(1 + \delta) + F_\epsilon(H + d - w)(3H + d - w)(1 + 2\delta) + \delta^2(H + d) - \delta(4H + d - w) - (3H + d - w)}{(1 + \delta)^2}$$

$$U_H = F_\epsilon(H + d + w)(H + (H - w)x_H^*)$$

$$= \frac{F_\epsilon(H + d + w)(3H + d - w)\delta(1 + \delta) - F_\epsilon(H + d - w)(3H + d - w)(1 + 2\delta) - \delta^2(H + d) + \delta(4H + d - 2w) + 3H + d - w}{(1 + \delta)^2}$$

Subgame: bargain with L first, H second

Second bargain with H: dictator proposes x_H to keep $1 - x_H$; elite proposes y_H to keep $1 - y_H$; these are proportions of the benefit from defeating the other elite L. If they do not reach an agreement, no conflict

$$U_D(\text{accept}) = F_\epsilon(H + d + w)(H + d + (H - w)y_H)$$

$$U_D(\text{reject}) = \delta F_\epsilon(H + d + w)(H + d + (H - w)(1 - x_H)) + (1 - \delta)(H + d)$$

$$U_H(\text{accept}) = F_\epsilon(H + d + w)(H + (H - w)x_H)$$

$U_H(\text{reject}) = \delta F_\epsilon(H + d + w)(H + (H - w)(1 - y_H)) + (1 - \delta)(H)$ if H doesn't come to an agreement with the dictator, no conflict

$$y_L^* = \frac{F_\epsilon(H + d + w)(\delta(2H - w) - d - H) - \delta H + H + d}{F_\epsilon(H + d + w)(H - w)(1 + \delta)}$$

$$x_L^* = \frac{F_\epsilon(H + d + w)(\delta(2H + d - w) - H) - \delta(H + d) + H}{F_\epsilon(H + d + w)(H - w)(1 + \delta)}$$

$$U_D = F_\epsilon(H + d + w)(H + d + (H - w)(1 - x_H^*))$$

$$= \frac{F_\epsilon(H+d+w)(3H+d-w)+\delta(H+d)-H}{1+\delta}$$

$$U_H = F_\epsilon(H + d + w)(H + (H - w)x_H^*)$$

$$= \frac{F_\epsilon(H+d+w)\delta(3H+d-w)-\delta(H+d)+H}{1+\delta}$$

first bargain with L: dictator proposes x_L to keep $1 - x_L$; elite proposes y_L to keep $1 - y_L$; these are proportions of the benefit from defeating the other elite H; if they don't reach an agreement, the dictator will work with H and fight L

$$U_D(\text{accept}) = F_\epsilon(H + d - w)(H + d + Hy_L)$$

$$U_D(\text{reject}) = \delta(F_\epsilon(H + d - w)(H + d + H(1 - x_L))) + (1 - \delta)\left(\frac{F_\epsilon(H+d+w)(3H+d-w)+\delta(H+d)-H}{1+\delta}\right)$$

$$U_L(\text{accept}) = F_\epsilon(H + d - w)(H - w + Hx_L)$$

$$U_L(\text{reject}) = \delta(F_\epsilon(H + d - w)(H - w + H(1 - y_L))) + (1 - \delta)(1 - F_\epsilon(H + d + w))(3H + d - w)$$

$$y_L^* = \frac{F_\epsilon(H+d+w)(3H+d-w)(1+\delta+\delta^2)+F_\epsilon(H+d-w)(\delta^2(2H-w)+\delta(H-d-w)-H-d)-\delta^2(3H+d-w)-\delta(2H-w)-H}{F_\epsilon(H+d-w)H(1+\delta)^2}$$

$$x_L^* = \frac{F_\epsilon(H+d-w)(\delta^2(2H+d)+\delta(H+d+w)-H+w)-F_\epsilon(H+d+w)(3H+d-w)(1+2\delta)-\delta^2(H+d)+\delta(4H+d-w)+3H+d-w}{F_\epsilon(H+d-w)H(1+\delta)^2}$$

$$U_D = F_\epsilon(H + d - w)(H + d + H(1 - x_L^*))$$

$$= \frac{F_\epsilon(H+d+w)(3H+d-w)(1+2\delta)+F_\epsilon(H+d-w)(3H+d-w)(1+\delta)\delta^2(H+d)-\delta(4H+d-w)-(3H+d-w)}{(1+\delta)^2}$$

$$U_L = F_\epsilon(H + d - w)(H - w + Hx_L^*)$$

$$= \frac{F_\epsilon(H+d-w)(3H+d-w)\delta(1+\delta)-F_\epsilon(H+d-w)(3H+d-w)(1+2\delta)-\delta^2(H+d)+\delta(4H+d-w)+(3H+d-w)}{(1+\delta)^2}$$

Which order does he prefer in this subgame?

$$U_D(L, H) - U_D(H, L) = \frac{(F_\epsilon(H+d+w)-F_\epsilon(H+d-w))((3H+d-w)+w)\delta}{(1+\delta)^2}$$

This is positive (or 0) because of non-decreasing proper CDF. Therefore if the dictator's post-

bargain preference is for no conflict (uncertainty is sufficiently high), he prefers to choose the bargaining order L, then H. He comes to an agreement with L and they both fight H.

$$[x_L^* = \frac{F_\epsilon(H+d-w)(\delta^2(2H+d)+\delta(H+d+w)-H+w)-F_\epsilon(H+d+w)(3H+d-w)(1+2\delta)-\delta^2(H+d)+\delta(4H+d-w)+3H+d-w}{F_\epsilon(H+d-w)H(1+\delta)^2}]$$